γ from $B \to DK$

Jure Zupan

Carnegie Mellon Univ. & Univ. of Ljubljana

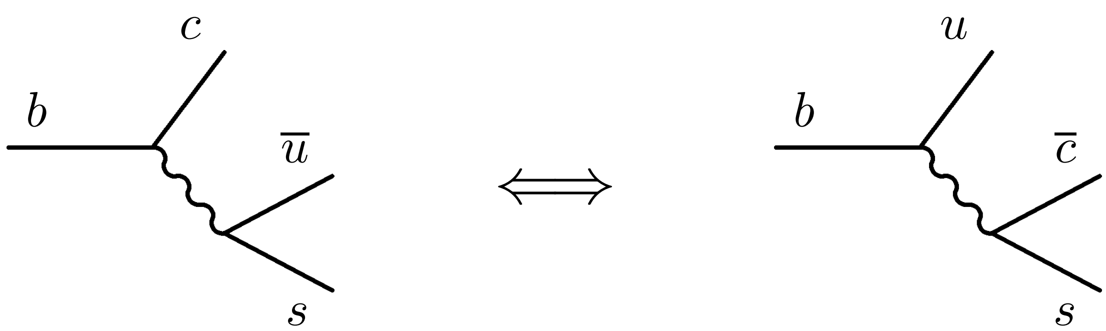
Outline

- short overview
- Dalitz plot analysis
- charm factory input
- theory errors
  - the effect of $D - \bar{D}$ mixing
- conclusions

Obtaining γ

- use interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$

Gronau, Wyler, 1991



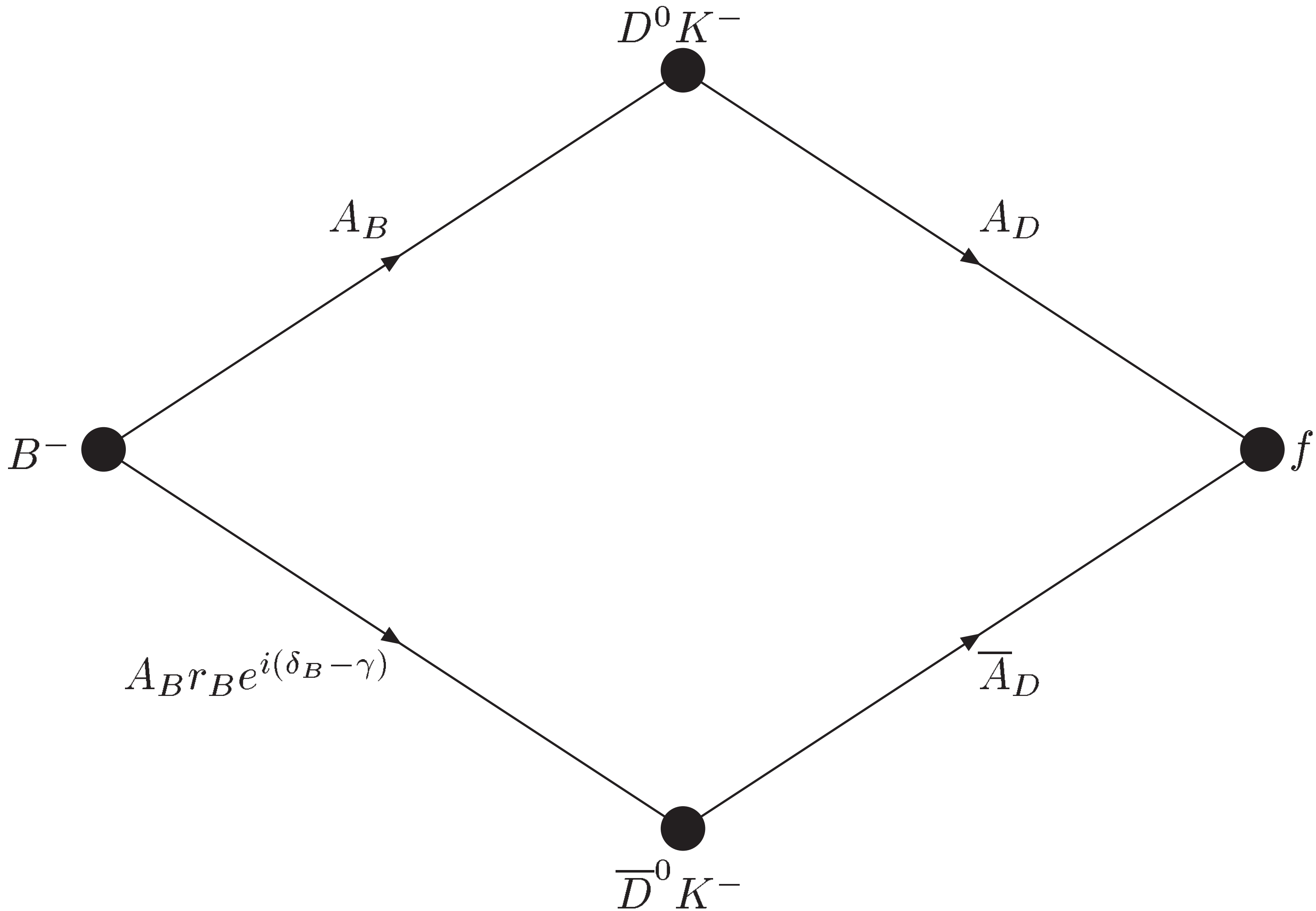
interference between

$B^- \to DK^-$	followed by	$D \to f$
$B^- \to \bar{D}K^-$	followed by	$\bar{D} \to f$

with f any common final state of D and $\bar{D}$

- no penguin contributions

graphically...



Different methods

methods can be grouped by the choice of final state f

- CP- eigenstate (e.g. $K_S\pi^0$) Gronau, London, Wyler (1991)
- flavor state (e.g. $K^+\pi^-$) Atwood, Dunietz, Soni (1997)
- singly Cabibbo suppressed (e.g. $K^{*+}K^-$) Grossman, Ligeti, Soffer (2002)
- many-body final state (e.g. $K_S\pi^+\pi^-$) Giri, Grossman, Soffer, JZ (2003)

other extensions:

- many body B final states (e.g. $B^+ \to DK^+\pi^0$) Aleksan, Petersen, Soffer (2002)
- use D^{0*} in addition to D^0
- use self tagging D^{0**} Sinha (2004)
- neutral B decays (time dependent and time-integrated) many refs.

Combining methods

- all methods are statistic dominated at present
- for different D channels the B system parameters are common

combining different D channels buys you more than just statistics

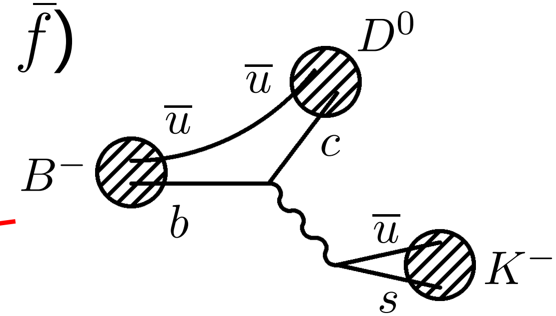
cf. talk by S. t'Jampens on monday

Counting the parameters

as an example consider the decay chain

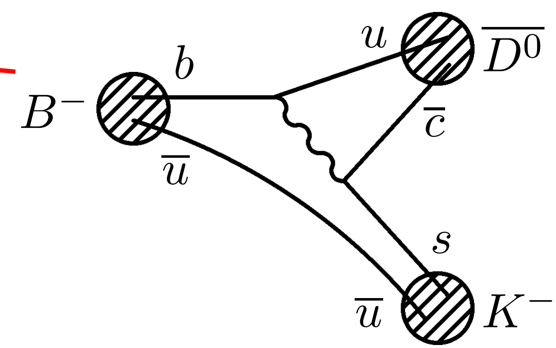
$$B^\pm \to DK^\pm \to (f)_D K^\pm \; (f \neq \bar{f})$$

the amplitudes for B decays



$$A(B^- \to D^0 K^-) \equiv A_B$$

$$A(B^- \to \overline{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}$$



color suppression + CKM $\Rightarrow r_B \sim 0.1$

neglecting CP violation in $D^0 \to f$ decay:

$$A(D^0 \to f) = A(\bar{D}^0 \to \bar{f}) = |A_f|$$

$$A(\bar{D}^0 \to f) = A(D^0 \to \bar{f}) = |\bar{A}_f| e^{i\delta_f}$$

Counting the parameters

- unknowns:
  - common to all decays: γ
  - $3N_B$ from the B system: A_B, r_B, δ_B for each $B \to DK, D^*K, DK^*$
  - N_f from the D system: δ_f ($|A_f|$, $|\bar{A}_f|$ measured)
- measurables (from B decays):
  - $B^\pm \to f_D K^\pm$, $B^\pm \to \bar{f}_D K^\pm$ decay widths
- $3N_B + N_f + 1$ unknowns vs. $4N_f N_B$ measurements
  - solvable for $N_B \geq 1$, $N_f \geq 2$
- slightly different counting if some $f = f_{CP}$ or semilept.

three body D decays

- the extreme case: $B^\pm \to [K_S\pi^+\pi^-]_D K^\pm$, where a continous set of "channels" in three-body final state

$$A_f \to A_D(s_{12}, s_{13}) \equiv A_{12,13}\, e^{i\delta_{12,13}}$$
$$\equiv A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$

- even better: with some model dependence one can measure also the variation of the phase $\delta_{12,13}$ over the Dalitz plot $\Rightarrow A_f$ completely known (!)
- with higher statistics a model independent treatment also possible

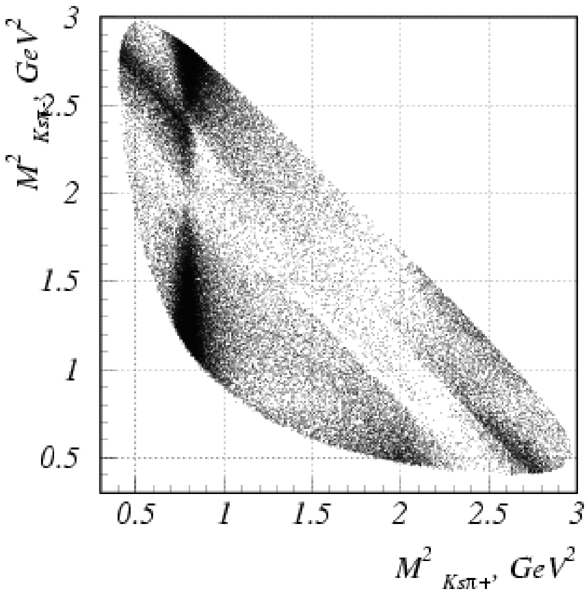
Modeling A_D

model A_D with a fit to a sum of Breit-Wigners:

$$A_D(s_{12}, s_{13}) = A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) =$$

$$= a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

$$\mathcal{A}_r(s_{12}, s_{13}) = {}^J\mathcal{M}_r \times \frac{1}{s - M_r^2 + iM_r\Gamma_r(\sqrt{s})}$$



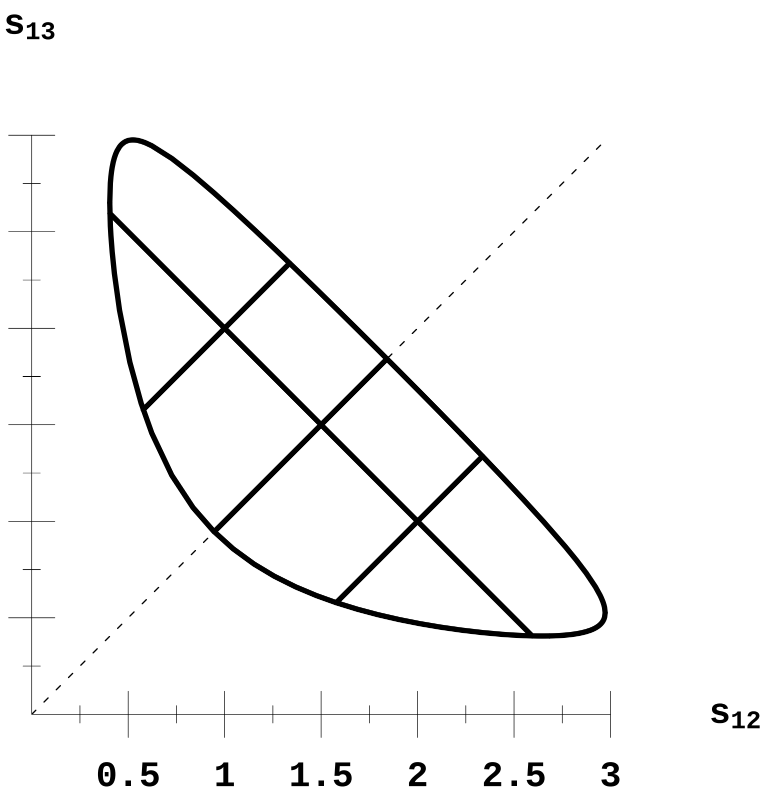
- model fit to high statistics D^0 tagged decay data
- in $B^\pm \to (K_S\pi^-\pi^+)_D K^\pm$ only r_B, δ_B and γ to be fit
- Q: what is the modelling error?

Belle, hep-ex/0308043

Model independent method

Giri, Grossman, Soffer, JZ, 2003

- at present the modeling error on γ is estimated to $\sim 10^\circ$
- partition Dalitz plot in bins



- unknowns:

$$c_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \cos(\delta_{12,13} - \delta_{13,12})$$

$$s_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \sin(\delta_{12,13} - \delta_{13,12})$$

- using CP : $c_{\bar{i}} = c_i,\, s_{\bar{i}} = -s_i$
- as in two-body decays enough measurables

$s_{12} = m^2_{K_s\pi^-}$ and $s_{13} = m^2_{K_s\pi^+}$

Input from charm factory

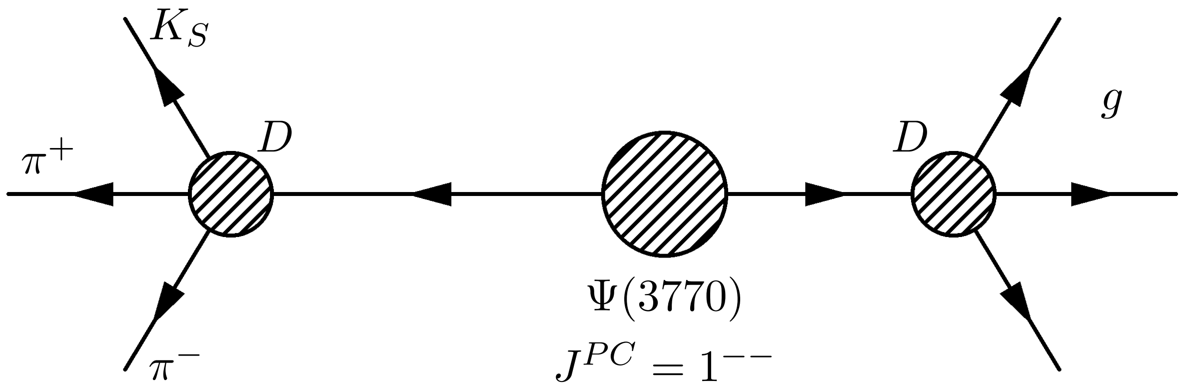
- charm factory can have a big impact
- working at $\psi(3770) \to D\bar{D}$ can measure $\arg(A_D)$

$\Rightarrow A_D$ completely known model indep.

- with this input one would be measuring from B decays only B-system parameters (like in the model-dependent Dalitz analysis)

Measuring phases @ charm factory

- entangled states: consider the following decay



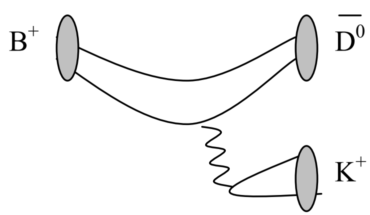
ith bin of $K_S\pi^+\pi^-$ and jth bin of g

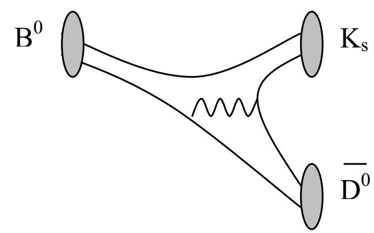
$$\Gamma_{i,j} \propto T_i T^g_{\bar{j}} + T_{\bar{i}} T^g_j - 2(c_i c^g_j + s_i s^g_j)$$

- if g a CP even (odd) eigenstate, $s^g_j = 0$, $T^g_j = T^g_{\bar{j}} = \pm c^g_j$, no sensitivity to s_i
- if $g = K_S\pi^+\pi^-$ and $j = i$ ($j = \bar{i}$) one measures s_i^2

Increasing statistics

- include as many D decay modes as possible
- include more B decay modes
  - $B^{\pm} \to D^* K^{*\pm} \Rightarrow$ would need to measure polarizations $\Rightarrow$ hard!
  - $B^0 \to D K_S$
- at first glance neutral B decays less attractive
  - have smaller decay rates: $A_n \sim \frac{1}{3} A_c$

$A_c =$ 

$A_n =$ 

  - time dependent measurements?

Gronau & London, 1990
Kayser & London, 1999

Using neutral B decays

- but: the statistical error on γ scales roughly as the smallest amplitude
- both in B^+ and B^0 these are color suppressed, using isospin (and negl. annih.):

$$A(B^+ \to D^0 K^+) \simeq \sqrt{2} A(B^0 \to D^0 K_S)$$

or

$$A_c r_c \simeq \sqrt{2} A_n r_n$$

- time integrated rates (untagged rates) of B^0 alone are sufficient to determine γ

Gronau, Grossman, Shumaher, Soffer, J.Z., (2004)

Almost there?

- BaBar analysis of $B^0 \to DK^{*0}$ hep-ex/0604016
- finds $r_n^* < 0.4$ @ 90% CL
- from isospin

$$r_n^* = \frac{A_c^*}{A_n^*} r_c^* = \sqrt{\frac{\Gamma(B^+ \to \bar{D}^0 K^{*+})}{\Gamma(B^0 \to \bar{D}^0 K^{*0})}} r_c^* \Rightarrow$$

$$r_n^* = (4.0 \pm 0.5) r_c^* = 0.4^{+0.3}_{-0.4}$$

(using $r_c^* = 0.11^{+0.08}_{-0.11}$ from S. t'Jampens's talk)

Theory errors

- $D - \bar{D}$ mixing the largest uncertainty (still very small!)
- in SM $D - \bar{D}$ mixing is CP conserving to very good approximation, $\theta \sim O(10^{-4})$
- the effect on γ is $O(x^2, y^2)$ Grossman, Soffer, JZ, 2005

  here $x \equiv \frac{\Delta m_D}{\Gamma_D}, y \equiv \frac{\Delta \Gamma_D}{2\Gamma_D}$, with $x \sim y \sim O(10^{-2})$

Effect of $D-\bar{D}$ mixing

- time-integr. decay rates measured in tagged D decays

$$\Gamma_f = \int dt|\mathcal{A}_f(t)|^2, \qquad \bar{\Gamma}_f = \int dt|\bar{\mathcal{A}}_f(t)|^2,$$

where $\mathcal{A}_f(t) = \mathcal{A}(D^0(t)\to f)$, $\bar{\mathcal{A}}_f(t) = \mathcal{A}(\bar{D}^0(t)\to f)$

- these are exactly the same as in B decay rate

$$\Gamma(B^+\to f_D K^+) \propto \bar{\Gamma}_f + r_B^2\Gamma_f + 2r_B\,\mathcal{R}e\Big(e^{i(\delta_B+\gamma)}\int dt\mathcal{A}_f(t)\bar{\mathcal{A}}_f(t)^*\Big)$$

- the effect of $D-\bar{D}$ mixing on the γ measurement only in the interference term

$$\int dt\mathcal{A}_f(t)\bar{\mathcal{A}}_f(t)^* \equiv \sqrt{\Gamma_f\bar{\Gamma}_f}e^{i\tilde{\delta}_f}e^{-\epsilon_f}$$

- $\tilde{\delta}_f$ is pure strong phase, does not effect γ extraction
- ϵ_f is dilution parameter

$D - \bar{D}$ mixing

- the dilution due to $D - \bar{D}$ mixing

$$\epsilon_f = \frac{1}{8}(x^2 + y^2)\left(\frac{1}{r_f^2} + r_f^2\right) - \frac{1}{4}\left(x^2 \cos 2\delta_f + y^2 \sin 2\delta_f\right)$$

  where $r_f = |A(\bar{D}^0 \to f)/A(D^0 \to f)|$

- it gives the approximate magnitude of the shift $\Delta\gamma$ in the determination of γ
- the leading term in ϵ_f is proportional to $(x^2 + y^2)/r_f^2$, $\Delta\gamma$ is larger for cases where r_f is smaller
- for DCS decays the shift $\Delta\gamma \lesssim 1^\circ$, other much smaller

Theory errors in the far future

- the effect of $D - \bar{D}$ mixing can be included once x, y are measured
- in model independent Dalitz plot analysis no changes needed
  - here one can fit for both δ_f and ϵ_f (since this equivalent to c_i, s_i)
- then the theory error is coming from
  - higher electroweak corrections
  - CP violation in D decays $\Rightarrow \Delta\gamma \sim O(x\theta, y\theta)$
- the error is conservatively $\Delta\gamma < 10^{-5}$
- will be statistics dominated for a long time

Conclusions

- γ extraction from $B^{\pm} \to DK^{\pm}$ offers the theoretically cleanest measurement of SM CKM phase
- for the foreseeable future the determination will be statistics dominated

Backup slides

Master formulae

- a set of $4k$ equations
- the k equations for i bins

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$
$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i]$$

eqs. for $\hat{\Gamma}_{\bar{i}}^-$, $\hat{\Gamma}_i^+$, $\hat{\Gamma}_{\bar{i}}^+$ obtained by $12 \leftrightarrow 13$ and/or $\gamma \leftrightarrow -\gamma$

- $2k+3$ unknowns: $c_i,\ s_i,\ r_B,\ \delta_B,\ \gamma$

solvable for $k \geq 2$

Comment on use of D^*

Bondar, Gershon (2004)

- define CP eigenstates

$$D^{(*)}_{CP=\pm} = \frac{1}{\sqrt{2}}\left(D^{0(*)} \pm \bar{D}^{0(*)}\right)$$

$$\begin{matrix} \text{CP}(\pi^0) = -1 \\ \text{CP}(\gamma) = +1 \end{matrix} \Rightarrow \begin{matrix} D^*_\pm \to D_\pm \pi^0 \\ D^*_\pm \to D_\mp \gamma \end{matrix}$$

- introduces a sign flip

$$Br(B^\pm \to D^*[D_f\pi^0]K^\pm) = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \pm \gamma)$$

$$Br(B^\pm \to D^*[D_f\gamma]K^\pm) = r_B^2 + r_D^2 - 2r_B r_D \cos(\delta_B + \delta_D \pm \gamma)$$

$$A_D = r_D e^{-i\delta_D}\bar{A}_D$$

Decay width

reduced partial decay width

$$d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) = \Big(A_{12,13}^2 + r_B^2\, A_{13,12}^2$$
$$+2r_B\mathcal{R}e\big[A_D(s_{12}, s_{13})\, A_D^*(s_{13}, s_{12})\, e^{-i(\delta_B-\gamma)}\big]\Big)dp$$

Untagged decays

$$\begin{aligned}
\langle\Gamma(B\to f_D K_n)\rangle &\equiv \Gamma(B^0\to f_D K_S)+\Gamma(\bar{B}^0\to f_D K_S)\\
&= A_f^2\left[X_n(1+r_f^2)+2Y_n r_f\cos(\delta_f+\gamma)\right]\\
\langle\Gamma(B\to \bar{f}_D K_n)\rangle &\equiv \Gamma(B^0\to \bar{f}_D K_S)+\Gamma(\bar{B}^0\to \bar{f}_D K_S)\\
&= A_f^2\left[X_n(1+r_f^2)+2Y_n r_f\cos(\delta_f-\gamma)\right]
\end{aligned}$$

where

$$X_n \equiv A_n^2(1+r_n^2)\ , \qquad Y_n \equiv 2A_n^2 r_n\cos\delta_n$$

Four hadronic unknowns: X_n, Y_n, δ_f and a weak phase γ