Signatures of strong-field QED effects in high-intensity laser-matter interactions

C. P. Ridgers
York Plasma Institute,
Department of Physics
University of York
Authors

- University of Strathclyde: R. Capdessus, W. Luo, M. J. Duff, P. McKenna, Z.-M. Sheng
- Central Laser Facility: A. P. L. Robinson
- Cockroft Institute/University of Lancaster: D. Seipt
- Cockroft Institute, University of Lancaster & University of Michigan: A. G. R. Thomas
- Chalmers University of Technology: T. G. Blackburn, M. Marklund
- Max Planck Institute for Nuclear Physics: J. G. Kirk
- Imperial College London: S.P.D. Mangles
How strong is a 'strong-field'? Introduction to QED model

Laser absorption by self-generated pair plasmas

Experiments we can perform now?

Spin polarisation of the plasma
Outline

- How strong is a 'strong-field'?
- Introduction to QED model
- Laser absorption by self-generated pair plasmas
- Experiments we can perform now?
- Spin polarisation of the plasma
Rapid Increase in Laser Intensity

- Intensity/Wcm$^2$
- Year

1960

1980

2000

2020

10$^{16}$

10$^{20}$

10$^{24}$

Rapid increase in laser intensity
Rapid Increase in Laser Intensity

- Intensity/Wcm⁻²
  - 10²⁴
  - 10²⁰
  - 10¹⁶

Year
- 1960
- 1980
- 2000
- 2020

Hercules (UMICH) → 2x10²² Wcm⁻²

Rapid increase in laser intensity
Rapid Increase in Laser Intensity

- **Hercules (UMICH)** → $2 \times 10^{22} \text{Wcm}^{-2}$

- Rapid increase in laser intensity

Graph showing the increase in laser intensity from 1960 to 2020, with a significant increase in intensity around the year 2000.
Rapid Increase in Laser Intensity

Intensity/Wcm$^{-2}$

Relativistic Plasma

Plasma

10$^{16}$

10$^{20}$

10$^{24}$

1960 1980 2000 2020

Year

Hercules (UMICH) → 2x10$^{22}$Wcm$^{-2}$

Rapid increase in laser intensity
Rapid Increase in Laser Intensity

Intensity/Wcm$^{-2}$

QED Plasma
$10^{24}$

Relativistic Plasma
$10^{20}$

Plasma
$10^{16}$

Year

1960 1980 2000 2020

Hercules (UMICH) → $2 \times 10^{22}$ Wcm$^{-2}$

Rapid increase in laser intensity
Rapid Increase in Laser Intensity

Year

1960  1980  2000  2020

Intensity/Wcm⁻²

10¹⁶

10²⁰

10²⁴

QED Plasma

Relativistic Plasma

Plasma

Rapid increase in laser intensity

Hercules (UMICH) → 2×10²² Wcm⁻²

APOLLON, ELI, Texas PW
Intensity to reach QED-plasma regime

Field does work \( \sim m_e c^2 \) over \( \lambda_c \)

\[
eE_s \lambda_c = m_e c^2
\]
Intensity to reach QED-plasma regime

Field does work $\sim m_e c^2$ over $\lambda_c$.

$$eE_s \lambda_c = m_e c^2$$

$$E_s = \frac{m_e c^2}{e \lambda_c} = 1.3 \times 10^{18} \text{ Vm}^{-1}$$

$$I_s = 2 \times 10^{29} \text{ Wcm}^{-2}$$
Intensity to reach QED-plasma regime

Field does work $\sim m_e c^2$ over $\lambda_c \rightarrow eE_s \lambda_c = m_e c^2$

$$E_s = \frac{m_e c^2}{e \lambda_c} = 1.3 \times 10^{18} \text{ Vm}^{-1}$$

$$I_s = 2 \times 10^{29} \text{ Wcm}^{-2}$$

Important parameter: $\eta = \frac{E_{RF}}{E_s}$  
Lab frame: $\eta = \frac{\gamma}{E_s} |E_\perp + \mathbf{v} \times \mathbf{B}|$
Intensity to reach QED-plasma regime

Field does work $\sim m_e c^2$ over $\lambda_c \implies eE_s \lambda_c = m_e c^2$

$$E_s = \frac{m_e c^2}{e \lambda_c} = 1.3 \times 10^{18} \, \text{Vm}^{-1}$$

$$I_s = 2 \times 10^{29} \, \text{Wcm}^{-2}$$

Important parameter: $\eta = \frac{E_{RF}}{E_s}$  

Lab frame: $\eta = \frac{\gamma}{E_s} \left| E_\perp + v \times B \right|$  

Assume $\gamma \sim a$  

$$a = \frac{e E_{\text{laser}}}{m_e c \omega_{\text{laser}}} \left| E_\perp + v \times B \right| \sim E_{\text{laser}}$$
Intensity to reach QED-plasma regime

Field does work $\sim m_e c^2$ over $\lambda_c \rightarrow eE_s \lambda_c = m_e c^2$

$$E_s = \frac{m_e c^2}{e \lambda_c} = 1.3 \times 10^{18} \text{Vm}^{-1}$$

$$I_s = 2 \times 10^{29} \text{Wcm}^{-2}$$

Important parameter: $\eta = \frac{E_{RF}}{E_s}$

Lab frame: $\eta = \frac{\gamma}{E_s} |E_\perp + v \times B|$  

Assume $\gamma \sim a$  

$$a = \frac{e E_{laser}}{m_e c \omega_{laser}}$$

$$|E_\perp + v \times B| \sim E_{laser}$$

Putting these into formula for $\eta$

$$\eta \sim 0.1 \frac{I}{5 \times 10^{22} \text{Wcm}^{-2}}$$
Why are strong-field interactions with matter interesting?

1. Applications – compact particle accelerator
Why are strong-field interactions with matter interesting?

1. **Applications** – compact particle accelerator

Ion acceleration by light pressure?
Why are strong-field interactions with matter interesting?

1. **Applications** – compact particle accelerator

Ion acceleration by light pressure?

Light pressure important at $>10^{23}\text{Wcm}^{-2}$
2. **Fundamental physics**

'Strong-field interaction': very non-linear Compton scattering.
2. **Fundamental physics**

'Strong-field interaction': very non-linear Compton scattering.

Particle basis states 'dressed' by background fields
Why are strong-field interactions with matter interesting?

2. **Fundamental physics**

'Strong-field interaction': very non-linear Compton scattering.

Particle basis states 'dressed' by background fields

Only possible to determine basis states in a few special background fields → plane waves, constant crossed electric and magnetic fields
Why are strong-field interactions with matter interesting?

3. **New plasma regime** – non-linear QED processes occur in plasma
Why are strong-field interactions with matter interesting?

3. **New plasma regime** – non-linear QED processes occur in plasma

**FEEDBACK**
QED processes

Classical Plasma Physics
Why are strong-field interactions with matter interesting?

3. **New plasma regime** – non-linear QED processes occur in plasma

Feedback:
- QED processes
- Classical Plasma Physics
Quasi-classical model

1. Split EM field into 'low frequency' background (laser-fields) & 'high frequency' emitted (gamma-rays) components
1. Split EM field into 'low frequency' background (laser-fields) & 'high frequency' emitted (gamma-rays) components
2. 'Low frequency' fields are treated classically

C.P. Ridgers et al, J Comp Phys, 260, 273 (2014)
Quasi-classical model

1. Split EM field into 'low frequency' background (laser-fields) & 'high frequency' emitted (gamma-rays) components
2. 'Low frequency' fields are treated classically
3. Use strong-field QED – basis states dressed by fields – motion classical between emissions

C.P. Ridgers et al, J Comp Phys, 260, 273 (2014)
V.N. Baier, 'High Energy Processes in Aligned Single Crystals'
Quasi-classical model

1. Split EM field into 'low frequency' background (laser-fields) & 'high frequency' emitted (gamma-rays) components

2. 'Low frequency' fields are treated classically

3. Use strong-field QED – basis states dressed by fields – motion classical between emissions

4. Keep lowest order interactions between electrons, positrons, gamma-rays with classical low frequency fields

C.P. Ridgers et al, J Comp Phys, 260, 273 (2014)
V.N. Baier, 'High Energy Processes in Aligned Single Crystals'
Quasi-classical model

1. Split EM field into 'low frequency' background (laser-fields) & 'high frequency' emitted (gamma-rays) components
2. 'Low frequency' fields are treated classically
3. Use strong-field QED – basis states dressed by fields – motion classical between emissions
4. Keep lowest order interactions between electrons, positrons, gamma-rays with classical low frequency fields

Photon emission

Pair production

C.P. Ridgers et al, J Comp Phys, 260, 273 (2014)
V.N. Baier, High Energy Processes in Aligned Single Crystals
Electromagnetic Cascade

Results in exponential growth in number of pairs
Outline

- How strong is a 'strong-field'?
  Introduction to QED model

- Laser absorption by self-generated pair plasmas

- Spin polarisation of the plasma

- Experiments we can perform now?
Ion acceleration reduced by ‘pair cushion’

Target:
Al – e\textsuperscript{-} density $10^{30}\text{m}^{-3}$
Fully ionised
Semi-infinite

Laser:
Circular polarisation
$I=5\times10^{24}\text{Wcm}^{-2}$
Pulse duration $\sim30\text{fs}$
Wavelength = 1 micron
Spot size 3.3 microns

Ion acceleration reduced by ‘pair cushion’

Electron density

Laser intensity

Ion density

Ion acceleration reduced by ‘pair cushion’

Ion acceleration reduced by ‘pair cushion’

Radiation pressure of laser pulse accelerates ions...

Ion acceleration reduced by ‘pair cushion’

... but eventually generates an overdense pair plasma which absorbs the energy and curtails acceleration.

Radiation pressure of laser pulse accelerates ions...

Ion acceleration reduced by ‘pair cushion’

Ion acceleration reduced by ‘pair cushion’

Ion acceleration reduced by ‘pair cushion’

Initially laser pulse perfectly reflected in HB frame

Ion acceleration reduced by ‘pair cushion’

Initially laser pulse perfectly reflected in HB frame

After some time pair cascade results in critical density pair plasma…..

Ion acceleration reduced by ‘pair cushion’

Initially laser pulse perfectly reflected in HB frame

After some time pair cascade results in critical density pair plasma.....

.....which radiates energy and laser pulse not perfectly reflected

Ion acceleration or pair production?

Ion acceleration or pair production?

Pair plasma creation

Ion acceleration or pair production?

Pair plasma creation

Ion acceleration

D. Del Sorbo, et al., arXiv:1706.04153
Absorption curtails ion acceleration

FEEDBACK
QED processes

Classical Plasma Physics
Absorption curtails ion acceleration

FEEDBACK
QED processes

Classical Plasma Physics

Skin effect curtails absorption
How strong is a 'strong-field'? Introduction to QED model

Laser absorption by self-generated pair plasmas

Experiments we can perform now?

Spin polarisation of the plasma
Rapid Increase in Laser Intensity

Hercules (UMICH) → $2 \times 10^{22} \text{Wcm}^{-2}$

Rapid increase in laser intensity
Electron Beam – Laser Pulse Collisions

\[ \eta \approx 0.1 \frac{I}{5 \times 10^{22} \text{Wcm}^{-2}} \]
Electron Beam – Laser Pulse Collisions

\[ \eta \approx 0.1 \frac{I}{5 \times 10^{22} \text{Wcm}^{-2}} \]

Assume \( \gamma \approx a \)

\[ a = \frac{e E_{\text{laser}}}{m_e c \omega_{\text{laser}}} \]

\[ \left| E_{\perp} + v \times B \right| \approx E_{\text{laser}} \]
Electron Beam – Laser Pulse Collisions

\[ \eta \sim 0.1 \frac{1}{5 \times 10^{22} \text{Wcm}^{-2}} \]

Assume \( \gamma \approx a \)

\[ a = \frac{e E_{\text{laser}}}{m_e c \omega_{\text{laser}}} \]

\[ |E_\perp + v \times B| \sim E_{\text{laser}} \]
Electron Beam – Laser Pulse Collisions

\[ \eta \sim 0.1 \frac{I}{5 \times 10^{22} \text{Wcm}^{-2}} \]

Assume \( \gamma \sim a \) \( a = \frac{e E_{\text{laser}}}{m_e c \omega_{\text{laser}}} \)

\[ \left| E_{\perp} + v \times B \right| \sim E_{\text{laser}} \]

\[ \eta \sim 0.1 \frac{\gamma m_e c^2}{500 \text{MeV}} \frac{I}{10^{21} \text{Wcm}^{-2}} \]
Signatures of strong-field QED effects

\[ \eta \sim 0.1 \frac{\gamma m_e c^2}{500 \text{MeV}} \frac{I}{10^{21} \text{Wcm}^{-2}} \]

We can reach \(\sim 1\)GeV and \(10^{21}\)Wcm\(^{-2}\) so \(\eta \sim 0.1\)
Signatures of strong-field QED effects

\[ \eta \sim 0.1 \frac{\gamma m_e c^2}{500 \text{MeV}} \frac{I}{10^{21} \text{Wcm}^{-2}} \]

We can reach \( \sim 1\text{GeV} \) and \( 10^{21} \text{Wcm}^{-2} \) so \( \eta \sim 0.1 \)

Sufficient to study photon emission and quantum effects on radiation reaction
Signatures of strong-field QED effects

\[ \eta \sim 0.1 \times \frac{\gamma m_e c^2}{500 \text{MeV}} \times \frac{I}{10^{21} \text{Wcm}^{-2}} \]

We can reach \(~1\text{GeV} and 10^{21}\text{Wcm}^{-2}\) so \(\eta \sim 0.1\)

Sufficient to study photon emission and quantum effects on radiation reaction

1. Decrease in power radiated compared to classical
2. Probabilistic nature of emission
Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’…

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_{\gamma}^{\infty} \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’….

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_{\gamma}^{\infty} \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

Electrons leave a region of phase space by emitting a photon and losing energy

Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’….

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_{\gamma}^{\infty} \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

Electrons leave a region of phase space by emitting a photon and losing energy

Electrons with higher energy enter the region of phase space as they emit

Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’….

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_\gamma^\infty \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

…. we can get equations for the evolution of the mean and variance of the energy

Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’....

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_{\gamma}^{\infty} \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

.... we can get equations for the evolution of the mean and variance of the energy

\[ \frac{d\langle \gamma \rangle}{dt} = -\frac{\langle gP \rangle}{m_e c^2} \]
\[ \frac{d\sigma^2}{dt} = \frac{\langle S \rangle}{m_e c^4} - \frac{\langle \Delta \gamma gP \rangle}{m_e c^2} \]

Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’….

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_{\gamma}^{\infty} \lambda \rho f(p') \frac{p'^2}{p^2} dp' \]

…. we can get equations for the evolution of the mean and variance of the energy

\[ \frac{d\langle \gamma \rangle}{dt} = -\frac{\langle gP \rangle}{m_e c^2} \]

\[ \frac{d\sigma^2}{dt} = \frac{\langle S \rangle}{m_e^2 c^4} - \frac{\langle \Delta \gamma gP \rangle}{m_e c^2} \]

Classically synchrotron losses narrow the electron spectrum
Signatures of strong-field QED effects

Starting with a quantum ‘emission operator’…. 

\[ \frac{\partial f}{\partial t} = -\lambda f + \frac{\eta}{\gamma} \int_0^\infty \lambda p f(p') \frac{p'^2}{p^2} dp' \]

…. we can get equations for the evolution of the mean and variance of the energy

\[ \frac{d\langle \gamma \rangle}{dt} = -\frac{\langle gP \rangle}{m_e c^2} \]
\[ \frac{d\sigma^2}{dt} = \frac{\langle S \rangle}{m_e^2 c^4} - \frac{\langle \Delta \gamma gP \rangle}{m_e c^2} \]

Quantum synchrotron effects can broaden the spectrum

Classically synchrotron losses narrow the electron spectrum
Signatures of strong-field QED effects

Distribution

\begin{align*}
\text{Distribution} & \\
\gamma & \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \\
\gamma & \quad 1000 \quad 1500 \quad 2000 \quad 2500 \quad 3000 \quad 3500 \\
\text{Initial} & \\
\end{align*}
Signatures of strong-field QED effects

Distribution

Final - classical

Initial

$\gamma$
Signatures of strong-field QED effects

Distribution

Initial

Final - classical

Final - quantum

Initial
Signatures of strong-field QED effects

\[ <\gamma> \]

\[
\frac{d<\gamma>}{dt} = -\frac{\langle gP \rangle}{m_e c^2}
\]

Quantum

Classical
Signatures of strong-field QED effects

\[ \frac{d \langle \gamma \rangle}{dt} = -\frac{\langle gP \rangle}{m_e c^2} \]
\[ \frac{d \sigma^2}{dt} = \frac{\langle S \rangle}{m_e^2 c^4} - \frac{\langle \Delta \gamma gP \rangle}{m_e c^2} \]
Outline

- How strong is a 'strong-field'? Introduction to QED model
- Laser absorption by self-generated pair plasmas
- Experiments we can perform now?
- Spin polarisation of the plasma
Sokolov-Ternov effect

Electron radiating in a strong magnetic field - asymmetry in rate of spin flip.....
Sokolov-Ternov effect

Electron radiating in a strong magnetic field - asymmetry in rate of spin flip.....

.... electron spin polarizes anti-parallel to B-field (positrons parallel)
Sokolov-Ternov effect

Electron radiating in a strong magnetic field - asymmetry in rate of spin flip.....

.... electron spin polarizes anti-parallel to B-field (positrons parallel)

Observed in storage rings: 1GeV electron in a 10kG magnetic field - 92.4% probability of spin anti-alignment after 1 hour
Analogous effect in laser fields?

Analogous effect in laser fields?

Electron radiating in a rotating electric field - asymmetry in rate of spin flip.....

Analogous effect in laser fields?

Electron radiating in a rotating electric field - asymmetry in rate of spin flip.....

.... electron spin polarizes anti-parallel to $\mathbf{E} \times \mathbf{\beta}$ (positrons parallel)

Analogous effect in laser fields?

Electron radiating in a rotating electric field - asymmetry in rate of spin flip.....

.... electron spin polarizes anti-parallel to $E \times \beta$
(positrons parallel)

Can this occur on the relevant fs timescale?

Spin polarisation in laser fields

Spin polarisation in laser fields

90% of electrons spin polarised anti-parallel to $E \times \beta$

Signatures of spin polarisation

Is this important?

Signatures of spin polarisation

Is this important?

1. Modifies radiation reaction by ~20%

Signatures of spin polarisation

Is this important?

1. Modifies radiation reaction by ~20%

2. Modifies polarisation of photons by ~30%

Signatures of spin polarisation

Is this important?

1. Modifies radiation reaction by ~20%

2. Modifies polarisation of photons by ~30%

3. Applications of spin polarised plasmas and electron beams?

Conclusions

Strong field QED processes are predicted to play a major role in 10PW laser-plasma interactions
Conclusions

- Strong field QED processes are predicted to play a major role in 10PW laser-plasma interactions
- Signatures include: curtailing ion acceleration (reducing energy by up to 65%); spin polarisation of the plasma
Conclusions

- Strong field QED processes are predicted to play a major role in 10PW laser-plasma interactions
- Signatures include: curtailing ion acceleration (reducing energy by up to 65%); spin polarisation of the plasma
- These processes can be measured using current PW lasers (colliding with energetic electron beam). Signatures: reduction in power radiated, stochasticity
Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame
Ion acceleration reduced by ‘pair cushion’


Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy

\[ \epsilon = m_I c^2 \frac{2 \Pi}{1+2 \sqrt{\Pi}} \]

Efficiency

\[ \varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1+2 \sqrt{\Pi}} \frac{1+R}{2} \]
Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy
\[ \epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}} \]

Efficiency
\[ \varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2} \]

Both depend on
\[ \Pi = \frac{1 + R}{2} \cdot \frac{I}{5 \times 10^{23} \text{ Wcm}^{-2}} \left( \frac{\rho}{1 \text{ gcm}^{-3}} \right)^{-1} \]

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy

$$\epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}}$$

Efficiency

$$\varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2}$$

Both depend on

$$\Pi = \frac{1 + R}{2} \frac{I}{5 \times 10^{23} Wcm^{-2}} \left(\frac{\rho}{1 \text{ gcm}^{-3}}\right)^{-1}$$

Reflection coefficient in hole-boring frame important

Ion acceleration reduced by ‘pair cushion’

Ion acceleration reduced by ‘pair cushion’

Simulation – cascade off

Theory - R=1

Ion acceleration reduced by ‘pair cushion’

Simulation – cascade off

Theory - R=1

Simulation – cascade on

Theory - R<1

QED-PIC Codes

Push particles

Update E & B

E & B

Current

'Standard' PIC code
QED-PIC Codes

Particle energies

Update E & B

Push particles

Update E & B

E & B

Current

QED-PIC code
QED-PIC Codes

Update QED rate equations

Generate photon/pair?

YES

Generate photon/pair

NO

Push particles

Update E & B

Particle energies

E & B

Generate photon/pair?

YES

Generate photon/pair

NO

Add particles to PIC code

Current

QED-PIC code
QED-PIC Codes

- Update QED rate equations
- Generate photon/pair?
  - YES: Generate photon/pair
  - NO: Push particles
    - Update E & B
    - E & B
    - Current
    - Add particles to PIC code
- QED-PIC code
-
- Monte-Carlo emission algorithm
Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy

$$\epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}}$$

Efficiency

$$\varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2}$$

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy

\[ \epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}} \]

Efficiency

\[ \varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \left( \frac{1 + R}{2} \right) \]

Both depend on \( \Pi = 0.2 \frac{I}{5 \times 10^{23} \, \text{Wcm}^{-2}} \left( \frac{\rho}{1 \, \text{gcm}^{-3}} \right)^{-1} \frac{1 + R}{2} \)

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy
\[ \epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}} \]

Efficiency
\[ \varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2} \]

Both depend on \( \Pi = 0.2 \)
\[ I = \frac{1}{5 \times 10^{23}} \frac{Wcm^{-2}}{Wcm^{-2}} \left( \frac{\rho}{1 gcm^{-3}} \right)^{-1} \frac{1 + R}{2} \]

Reflection coefficient in hole-boring frame important

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

\[
\begin{align*}
\epsilon &= m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}} \\
\varphi &= m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2}
\end{align*}
\]

Both depend on: \( \Pi = 0.2 \frac{I}{5 \times 10^{23} W cm^{-2}} \left( \frac{\rho}{1 g cm^{-3}} \right)^{-1} \frac{1 + R}{2} \)

\( R = 0 \) in weakly relativistic case \( \Pi \ll 1 \)

Gives 50% reduction in ion energy and 65% reduction in efficiency

Ion acceleration reduced by ‘pair cushion’

Analytical model for energy reduction – balance momentum in ‘hole boring’ frame

Ion energy

\[ \epsilon = m_I c^2 \frac{2 \Pi}{1 + 2 \sqrt{\Pi}} \]

Efficiency

\[ \varphi = m_I c^2 \frac{2 \sqrt{\Pi}}{1 + 2 \sqrt{\Pi}} \frac{1 + R}{2} \]

Both depend on \( \Pi = 0.2 \frac{I}{5 \times 10^{23} \text{ Wcm}^{-2}} \left( \frac{\rho}{1 \text{ gcm}^{-3}} \right)^{-1} \frac{1 + R}{2} \)

\( R = 0 \) In strongly relativistic case \( \Pi \gg 1 \)

Gives 30% reduction in ion energy and 50% reduction in efficiency