
*X-ray spectroscopic intercomparison between
hydrogenic ions of different Z as a method for
determining Lamb shifts*

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Introduction - the intercomparison technique

The 1s Lamb shift in medium and high Z hydrogenic ions remains an important test of QED theory in the strong field regime. As shown in the table, the experimental uncertainties in the best 1s Lamb shift measurements are considerably larger than the corresponding theoretical uncertainties. We show that high precision QED measurements can be made by comparing the wavelengths of transitions within hydrogenic ions of different Z.

Element	1s Lamb shift (eV)	Theoretical uncertainty (eV) [1]	Experimental Uncertainty (eV)
Ar	0.9383	7.4×10^{-4}	0.016 [2]
Ni	5.096	3.7×10^{-3}	0.10 [3]
Au	205.3	0.37	7.9 [4]
U	458.5	2.7	15 [4]

The energy of a level ($n, k=j+1/2$) of a hydrogenic ion of nuclear charge Z can be expressed as a sum of two parts, a large part given by the solution of the Dirac equation and believed to be exact,

$$E(n, k, Z) = \left(1 + \left[\frac{Za}{n - k + (k^2 - Z^2 a^2)^{1/2}} \right]^2 \right)^{-1/2} m_e c^2$$

and a much smaller non-Dirac part due to QED and finite nuclear size corrections (Lamb shift). The principle of the intercomparison technique is to choose a pair of transitions in a pair of hydrogenic ions of different Z for which the Dirac parts are nearly (or exactly) identical but whose non-Dirac parts are not. A measurement of the energy difference between such transitions is sensitive to the small uncertain part, and insensitive to the large, exact part.

The Z-2Z intercomparison

In equation 1, $E(an,ak,aZ) = E(n,k,Z)$ for any value of a . So it follows that the energy of the transition $2p_{1/2}-1s_{1/2}$ in the hydrogenic ion of nuclear charge Z has an identical Dirac part to that of $4d_{3/2}-2p_{3/2}$ in the hydrogenic ion of charge $2Z$. The energy difference between such transitions is entirely due to QED and nuclear size corrections. To illustrate the technique, figure 1 shows a simulation of the relevant transitions in Ar^{17+} and Kr^{35+} as expected from a crystal spectrometer on an EBIT source [5], as in the setup of figure 3. The linewidths are due to the spectrometer resolution, EBIT doppler broadening and the natural linewidths. A simulation of the ion densities trapped in the EBIT (as in [6]) allowed the line intensities to be calculated. In the figure, the marked interval has zero Dirac contribution as discussed.

The two y-scales of figure 1 indicate the difficulty with such a Z-2Z intercomparison - the Kr^{35+} lines are very much weaker than the Ar^{17+} lines. This arises because the electron impact excitation cross-sections scale as Z^4 and as n^{-3} and the 4d states receive only ~8% of the total $n=4$ population. By using data obtained with the setup of figure 3, and the results of the EBIT modelling code, we estimated the signal rate to be expected for the Kr^{35+} lines. We find that it is unfeasible to make a precision QED measurement by the Z-2Z technique and the setup of figure 1.

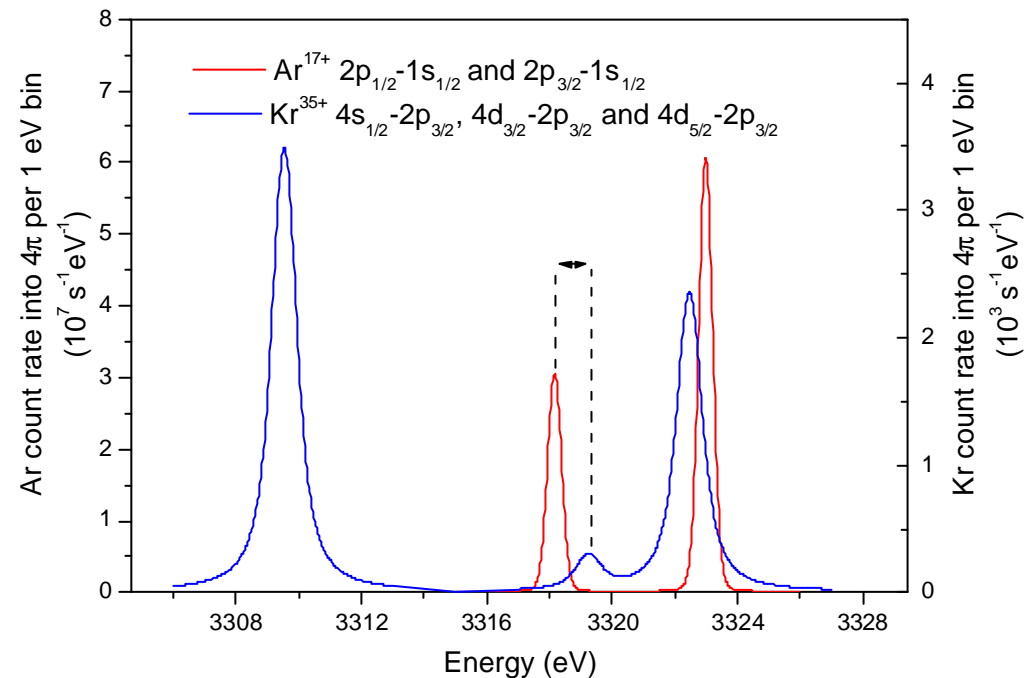


Figure 1

The Lyman series intercomparison - medium Z

A second technique, which avoids the difficulty of the unfavourable excitation cross-sections, involves an energy comparison between the 2-1 transitions in the hydrogenic ion Z_1 , and the $n-1$ transitions in the hydrogenic ion Z_2 , with $n > 2$ and $Z_1 > Z_2$. For some specific values of n , Z_1 and Z_2 , the two sets of transitions are very close in energy. The close coincidences can be found by requiring that $(3/4)Z_1^2 \approx (1-1/n^2)Z_2^2$. In this case, the intercomparison involves the ground states of both the hydrogenic ions and so measures the energy difference between their 1s Lamb shifts. Although the Dirac parts are very nearly cancelled in the intercomparison, the difference between the Lamb shifts remains large because of its faster Z scaling.

Figure 2 shows a simulation for an intercomparison between 3p-1s in V^{22+} and 2p-1s in Mn^{24+} . The simulation is for a crystal spectrometer on an EBIT, as in the setup of figure 3. The line intensities are estimates based on our experience with this setup, results from the EBIT modelling code [6] and the known excitation cross-sections [7] and decay branching ratios. We estimate that the difference between the Lamb shifts of the two ions could be measured to an accuracy of 3 meV (0.3%) during a 7 day EBIT run period.

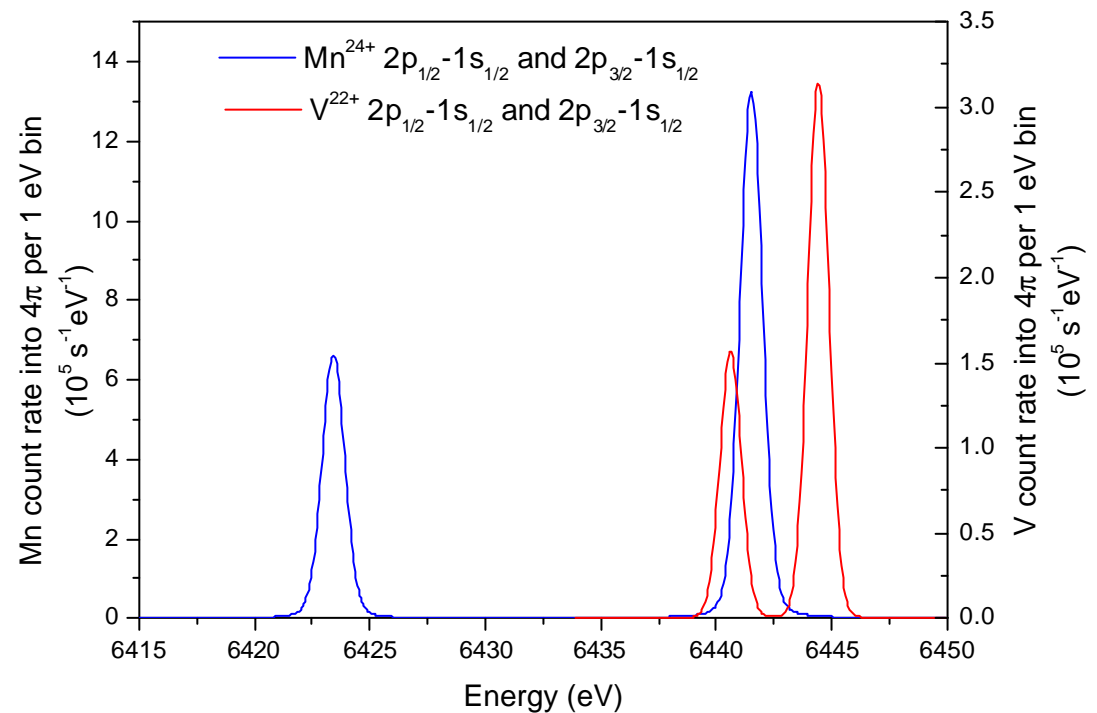


Figure 2

High resolution x-ray spectroscopy at the Oxford EBIT

At the Oxford EBIT [8] we have been using a curved crystal x-ray spectrometer in the Johann geometry, as shown in Figure 3. X-rays from the EBIT are Bragg reflected by the crystal to a position sensitive detector on the Rowland circle. The detector was a liquid nitrogen cooled x-ray CCD with $22.5\ \mu\text{m}$ spatial resolution and 150 eV energy resolution in every singly-occupied pixel. When the EBIT is inside the Rowland circle, a range of wavelengths can be reflected from crystal to detector without rotating the crystal.

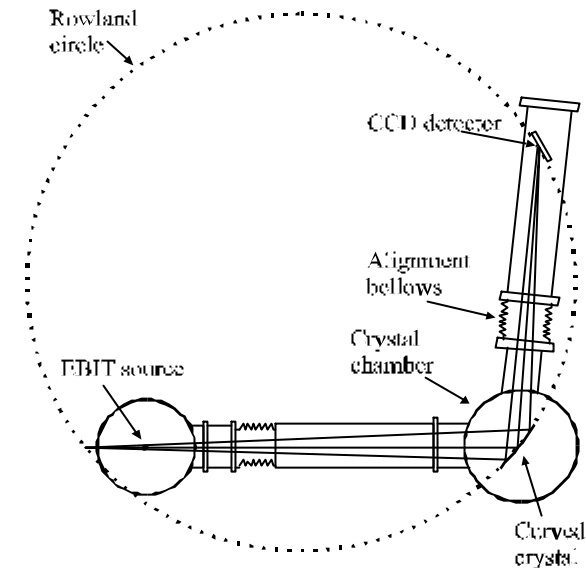


Figure 3

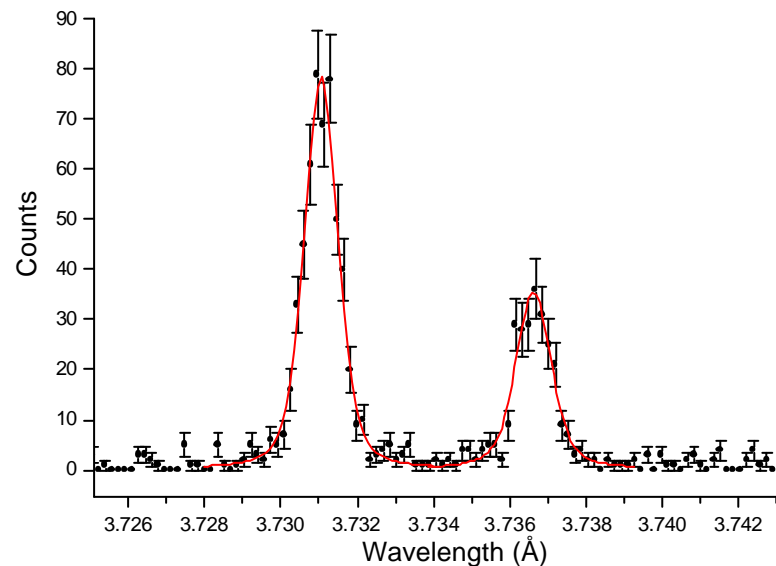


Figure 4

An example spectrum obtained with this setup is shown in Figure 4. It shows the Lyman- α lines of Ar^{17+} obtained in a 1-hour exposure using an electron beam current of 50 mA. The red line shows Voigt function fits to the data points. The resolution is $\Delta\lambda/\lambda=1/3500$. The detection efficiency of the spectrometer, including solid angle, was approximately 10^{-7} , and the total flux of Ar^{17+} Lyman- α x-rays was $\sim 3 \times 10^6\ \text{s}^{-1}$.

The Lyman series intercomparison - high Z

For the high Z hydrogenic ions, the flux of x-rays from an EBIT source is extremely small. We consider instead the use of a storage ring equipped with an electron cooler. The Doppler tuned absorption edge technique [9] can be used to measure the x-ray spectrum. Figure 5 shows a simulation of the Lyman- α lines of U^{91+} together with the Lyman- β lines of At^{84+} , Rn^{85+} and Fr^{86+} as they might appear if excited at the gas-jet target of the ESR at GSI. In this simulation, the measurements are made by Doppler tuning the transitions across the K-edge of a Dy absorber placed in front of a segmented Ge detector situated at a backward angle of 160° to the direction of the ~ 200 MeV/u beam. The Doppler tuning can be achieved by sweeping the beam energy via the cooler voltage.

The line intensities are those expected from the radiative electron capture of 10^8 stored ions in a nitrogen gas-jet of thickness 5×10^{11} cm $^{-2}$. The linewidths are 80 eV due to the width of the K-edge and lifetime broadening. The use of a large backward angle and segmented Ge detector ensure that the angle-induced Doppler broadening is small. Because the efficiency of the Doppler tuned technique is high, we estimate that such a high Z intercomparison measurement could reach a precision of 1 eV.

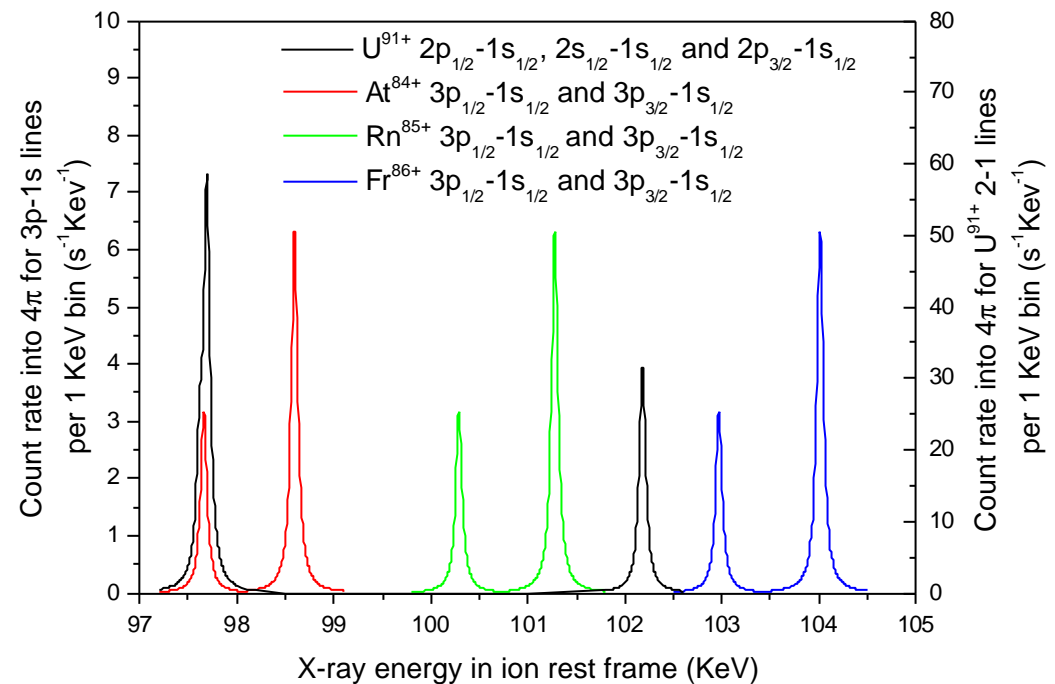


Figure 5

Acknowledgements and references

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