British Physics Olympiad

Paper 3. 2003.

Monday 3rd March 2003.

Time allowed 3hrs plus 15 minutes reading time.
All questions should be attempted. Question 1 carries 40 marks, the other questions 20 marks each.

Speed of light in free space	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Elementary charge	e	1.60×10^{-19}	C
Mass of proton	m_p	1.67262×10^{-27}	kg
Mass of neutron	m_n	1.67493×10^{-27}	kg
Mass of electron	m_e	9.11×10^{-31}	kg
Gravitational constant	G	6.67×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Boltzmann constant	k	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
Earth-Moon distance	R_{EM}	3.82×10^{8}	m
Radius of the Earth	R_{E}	6.37×10^{6}	m
Mass of the Sun	M_{Sun}	1.99×10^{30}	kg
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Relative Molecular mass of Oxygen		32	
Relative Molecular mass of Nitrogen		28	

Q1

In this question you are asked to make reasoned estimates and assumptions. These must be clearly stated.

a)

(i) On a really dark moonless night the planet Venus is sufficiently bright to produce shadows of objects on the Earth. Explain why the shadows are different to those produced by the Sun and the Moon. [2]

(ii) It is important not to under inflate car tyres – they could overheat and explode. Why is this? [2]

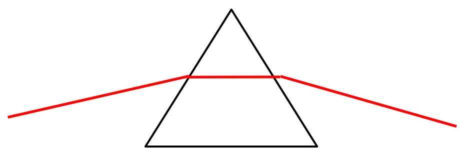
(iii) It is difficult to open small plastic bags for vegetables in the supermarket. A sharp blow along the edge in the plane of the bag usually works. Why? [2]

(iv) How long will it take to hard boil an ostrich egg? Explain how you reached your answer. [3]

[9]

(b)

Figure 1.1



(i) Figure 1.1 shows an equilateral glass prism illuminated by a 100 W laser beam of wavelength $\lambda = 600\,\text{nm}$. The refractive index of the glass of the prism is 1.50 at $\lambda = 600\,\text{nm}$. The path of the light in the prism is parallel to the base of the prism. The momentum of a photon is h/λ. Calculate the change in weight of the prism when the beam is switched on. [4]

(ii) Optical tweezers, which are composed of two lasers beams, are able to manipulate small transparent spheres. Explain clearly how this can be done and why at least two beams are needed. [3]

(iii) Small smoke particles in air are seen under a low magnification microscope to move randomly at a speed of $0.10\,\text{mm s}^{-1}$. The speed of sound in air is $330\,\text{m s}^{-1}$. Estimate the mass of the smoke particles. [3]

[10]

(c)

(i) Show that the de Broglie wavelength, λ, of a particle of mass m moving at a velocity of v where $v << c$ is related to the KE of the particle by:

$$\lambda = \frac{h}{\sqrt{2m.KE}}.$$

[3]

(ii) Consider the particle in a small rectangular box with sides of length *a, b* and *c*. The particle is moving at right angles to the "*b-c*" plane. Find an expression for the smallest possible energy of the particle. [3]

(Think about the amplitude of the wave at the wall of the box)

(iii) The box now contains many particles, and one "*b-c*" plane of the box is replaced by a piston. Show that as the length, *a*, is *slowly* decreased the resulting change in wavelength ensures that Boyle's Law is obeyed

[3]

[9]

d)

If an isotope of nitrogen is bombarded by alpha particles, protons are observed. The reaction that occurs is:

$$^{14}_{7}N + \alpha \rightarrow {}^{17}_{8}O + p \, .$$

Calculate the minimum K E of the alpha particle if this reaction is to occur. Explain your answer.

Data

Nuclei	*Mass/10^{-27}kg*
Nitrogen 14	23.2466
Oxygen 17	28.2209
Alpha particle	6.6442

[4]

e) A man is fishing in a boat floating in a small pond. To stop him drifting in the wind he takes a large rock, to which a rope is attached from inside the boat, and lowers the rock to the bottom of the pond, fastening the rope to the boat. Explain what happens to the level of the water in the *pond*?

[4]

f) Figure 1.2

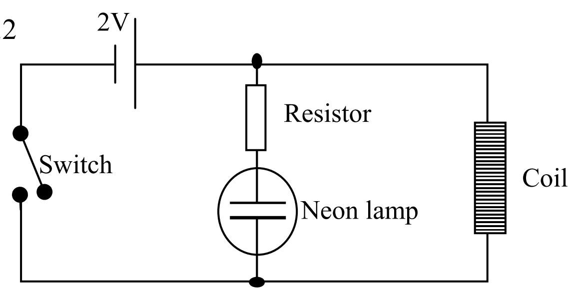


Figure 1.2 shows a circuit containing a resistor, coil, neon lamp, switch and 2 V cell. The bulb needs 80 V to light it and goes out if the PD across it drops below 60 V. The switch is closed and nothing is observed. The switch is opened and the Neon light flashes once. Explain. **[4]**

[40]

Q2. Missing mass.

Figure 2.1

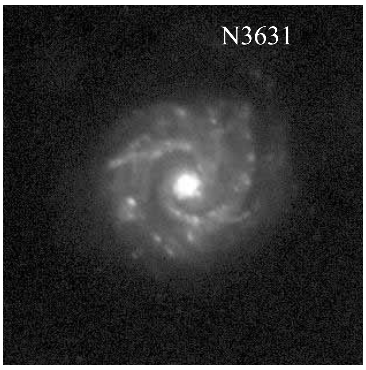


Figure 2.2

Figure 2.1 shows a photograph of the galaxy N3631. The axis of rotation of this disc-like galaxy is at right angles to the plane of the photograph. The velocity of stars in similar galaxies, that are seen edge on, can be determined from the Doppler shift of the spectral lines of the constituent stars. The rotation rates are slow and the speeds of the stars are very much smaller than the speed of light. Figure 2.2 shows a photo of the Coma cluster of galaxies. This contains at least one roughly spherical galaxy. The magnification of the Coma cluster is less than that of N3631.

a) A thin uniform density *spherical* shell is of mass M, radius R. Show that there is no gravitational field due to the shell inside the volume enclosed by the shell.

[4]

b) In the case of a spherically symmetrical body of uniform density and of radius R show that the gravitational field inside the body at a radius of r depends only on the mass of the body contained within radius r for $r \leq R$. Sketch graphs showing how you would expect (i) the gravitational field (ii) the gravitational potential to vary from the centre to a distance of $3R$ from the centre. [4]

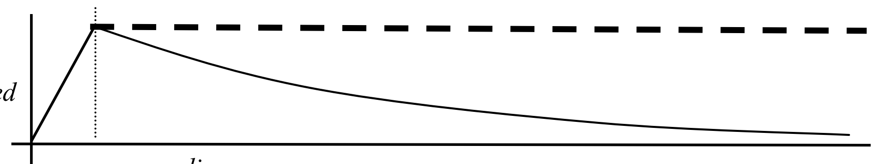
c) A ring of material has radius x. Explain why the gravitational field is not generally zero inside the plane of the ring and hence that the orbit of a star in the galaxy N3631 will be affected by the extent of the galaxy outside the orbit of the star. [4]

d) An estimate of the mass of the Coma cluster can be made from the velocity of the galaxies relative to the centre of mass of the cluster; the velocities are around $2000\,\mathrm{km\,s^{-1}}$. The stars that form the cluster have been together for a long time. Show that a simple model will predict a mass of the cluster of the order of 2×10^{43} kg. The radius of the Coma cluster is about 10^{21}m. [4]

e) Measurements can be made of the luminosity of the Coma cluster and compared with that of the Sun. The observed luminosity of the cluster divided by its mass is of the order of a hundred times less than that of the Sun. The implication is that most of the mass in the cluster is dark i.e. too cold to emit detectable radiation - hence the term **missing mass**.
It has been suggested that another solution to the problem is a modification of Newton's Laws of motion for very small accelerations, such as the Sun's acceleration towards the centre of the Galaxy ($\approx 10^{-10}\,\mathrm{m\,s^{-2}}$). For a *spherical,* constant density, galaxy show that the tangential speed/distance graph from the centre point should be a curve like that in Figure 2.3. How would you modify Newton's second law so that the dashed line in Figure 2.3 is obtained for orbits of stars outside the main, visible, mass of the galaxy? [4]

Figure 2.3

Tangential speed



radius

[20]

Q3

a) Figure 3.1a

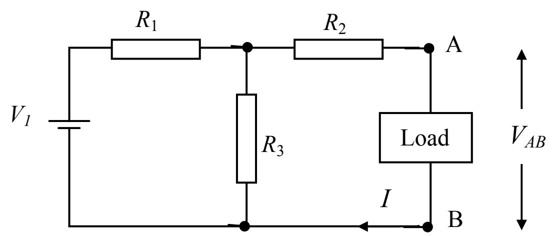


Figure 3.1b

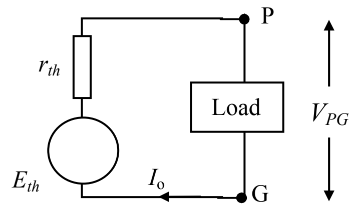


Figure 3.1a shows a circuit with three resistors, a load and one cell. Figure 3.1b shows a simplified circuit, where the three resistors have been replaced by a single resistor and the cell by a different cell. If $V_{AB}=V_{PG}$ and $I=I_o$. then show that this requires:

$$E_{th}=\frac{V_1R_3}{R_1+R_3} \text{ and } r_{th}=R_1+\frac{R_2R_3}{R_2+R_3}.$$

[7]

b) Figure 3.2

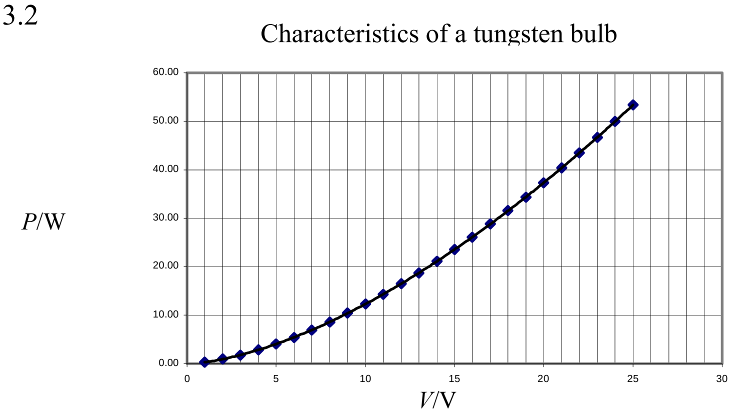


Figure 3.3

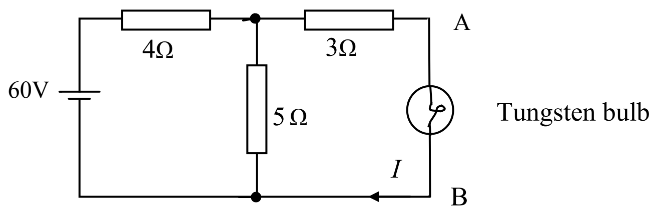


Figure 3.2 shows the power (P)/ P.D.(V) characteristics of a nominal 24 V, 50 W, bulb. These are the normal operating values. Figure 3.3 shows the circuit into which has been inserted. The equation of the curve in Figure 3.2 is $P=0.309V^{1.6}$. Use an equivalent circuit (as in 3.1b) to find the current I in the bulb and the power P dissipated in the bulb. [7]

If you use programmable functions on your calculator you must explain in detail your solution.

State and use simple assumptions to show that $P^5 \propto V^8$ is the expected relationship for a pure tungsten filament bulb. [6]

[20]

Q4

(a) Figure 4.1

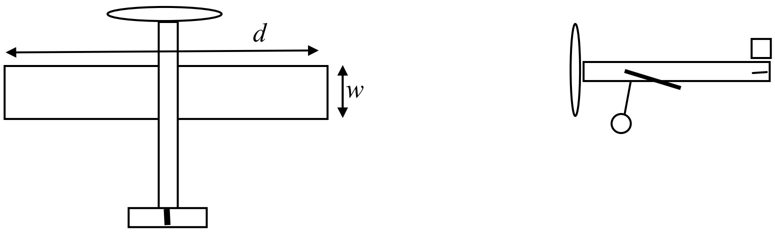


Figure 4.1 shows a diagram of a model aeroplane. The wing may be regarded as a thin rigid sheet; lift is obtained by the air hitting the angled wing. When the aeroplane is flying horizontally the wing is angled at 10° to the horizontal. The mass of the aeroplane is m and the density of air is ρ_{air}.

(i) Find an expression for v the speed of the aircraft, relative to the air, if it is to maintain horizontal flight.

(ii) Find the minimum power needed to maintain this speed if $m = 1$ kg, $\rho_{air} = 1.2$ kg m^{-2}, $d = 2$ m and $w = 0.2$ m.

(iii) Solar panels produce about 60 W m^{-2}. Comment on the wing area if the plane is to be driven by solar power alone. [8]

(b)

Figure 4.2

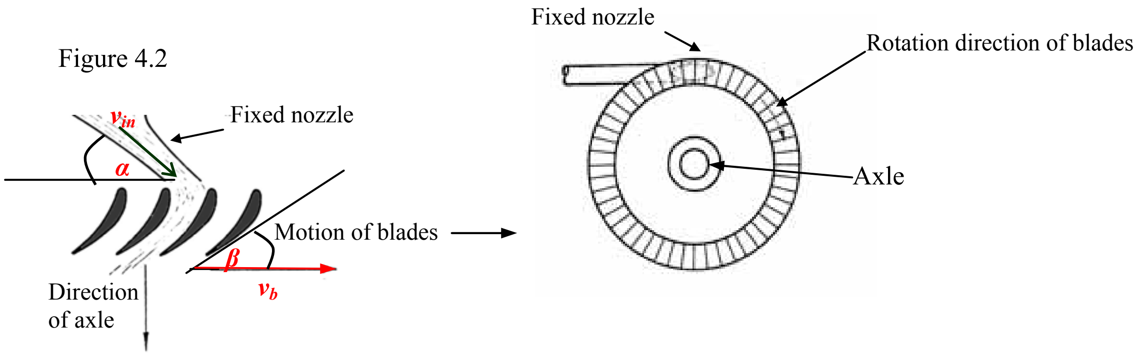


Figure 4.2 shows two views of a water turbine. The blades move at a mean tangential velocity v_b. The water from the fixed nozzle, velocity v_{in} makes an angle of α with the direction of the blades and the angle at which the water leaves the blades *relative to the blades* is β, Figure 4.3.

Figure 4.3

Copy the two vector diagrams in Figure 4.3 and complete the triangle of vectors. Explain the added vectors and their effects. [5]

(c)

Assume that the magnitude of the velocity of the water in the passage between the blades does not change. Write down an expression for the change in momentum of the water in the direction X in terms of v_{out}, v_{in}, α, β and the mass flow per second μ. Show that this leads to the expression:

$$Fv_b = \mu\left(\frac{v_{in}^{\,2} - v_{out}^{\,2}}{2}\right),$$ [5]

where F is the force on a blade. Explain the significance of this equation. [2]

[20]