Higher Criticism Statistic: Optimality and Applications in Cosmology and Astronomy

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Abstract

Higher Criticism is a new statistic first proposed by Donoho and Jin where it has been shown to be effective at resolving a very subtle testing problem: test whether $n$ normal means are all zero versus the alternative that a small fraction is non-zero.

Higher Criticism is also useful for non-Gaussian detection. Motivated by the recent study on the detection of cosmic strings (Jin et al. 2004), we consider a setting of nonGaussianity detection, in which we test whether $n$ independent tests are truly Gaussian, or each test is a superposition of a Gaussian component with a very faint nonGaussian component, whose distribution is unknown but symmetric and extremely heavy-tailed. We show that the Higher Criticism is asymptotically optimal for detection in such setting when the tail probability of nonGaussian component $\propto |x|^{-\alpha}$ and $\alpha < 8$, while the widely used statistic in cosmology - excess kurtosis - is asymptotically optimal when $\alpha > 8$.

As the above is also useful in the nonGaussian signature detection of the Cosmic Microwave Background (CMB). We have implemented the Higher Criticism in the first year data of the recent Wilkinson Microwave Anisotropy Project (WMAP, 2003), we found that the Higher Criticism is not only almost equally powerful as the kurtosis by reporting a nonGaussian detection with confidence > 99\%, but also suggests a way to track down the small portion of data corresponds to the nonGaussianity. Comparing to recent reported nonGaussianity detection (e.g. Vielva et al. 2004), our result suggests that a ring, not the whole spot, centered at (209°, −57°) in the WMAP corresponds to the nonGaussianity.

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