

LIMITS AND CONFIDENCE INTERVALS IN THE PRESENCE OF NUISANCE PARAMETERS

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We study the frequentist properties of confidence intervals computed by the method known to statisticians as the profile likelihood. It is seen that the coverage of these intervals is surprisingly good over a wide range of possible parameter values for important classes of problems, in particular whenever there are additional nuisance parameters with statistical or systematic errors. Programs are available for calculating these intervals.

We consider the problem of setting confidence limits for the signal rate in the presence of background which is estimated from data sidebands or Monte Carlo. Specifically we study the situation where the signal can be modeled by a Poisson distribution, a background with either a Poisson or a Gaussian distribution and an efficiency with either a Binomial or a Gaussian distribution. We establish the domain of validity, enabling comparison with other methods. We show that this method, together with some minor adjustments, has very good coverage even in cases when the parameters lie close to or at the physical boundaries.

Although this paper, and the corresponding routines, only deal with the specific problems outlined above, the results show that the method of profile likelihood is a viable technique for dealing with nuisance parameters, and it should be useful for other problems as well.

A stand-alone FORTRAN routine for calculating the limits discussed here is available at <http://charma.uprm.edu/~rolke/publications.htm>. It is also available as TRolke which is part of the ROOT system. Both routines also allow the calculation of the experimental sensitivity. Finally, at least for the cases where there are more events in the signal region than are expected from background, one could use MINUIT/MINOS to carry out the calculations, though in this case care needs to be taken to set the limits on the parameters correctly.

It is to be hoped that the profile likelihood method yields good results also in situations other than the ones discussed here. Because it is already available as part of MINUIT, its implementation for

different problems should be quite straightforward. It needs to be emphasized, though, that the profile likelihood method can not be assumed to yield good results in all cases and that it might require some adjustments to the general method as we have done here. It is therefore strongly recommended that a thorough check of its performance be done whenever it is applied to a new problem. In the case of setting limits, this means a coverage study as described above, at least for the range of likely parameter values.

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