Black holes and extra dimensions

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Motivation

**Black holes**: most interesting and intriguing solutions of Einstein's equations

**Extra dimensions** seem to be necessary in an ultimate theory of high energy physics

**Brane world models** → large extra dimensions

Higher dimensional black holes as classical solutions

**Black holes in accelerators!**
Outline

Black holes – basics

Extra dimensions – basics

Higher dim. black holes in brane world models
  i) black holes in accelerators - LHC
  ii) black holes by cosmic rays - Auger Observatory

Problem: ultra-fast proton decay

Realistic models with stable proton:
Black holes will NOT be observed at the Auger Observatory
Black Holes

Laplace in 18th century, Newtonian mechanics:

- Particle rotating around a massive object

\[ E_g = G \frac{mM}{r} = \frac{1}{2} mc^2 \quad \rightarrow \quad r_g = \frac{2GM}{c^2} \]

- Laplace called them “Dark Stars”
• Einstein, 1915, General Relativity

• Schwarzschild metric, 1916:

\[ ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega \]

\[ r = r_{\text{horizon}} = 2GM/c^2 \rightarrow \text{metric has coordinate singularity} \]

at \( r = r_{\text{horizon}} \) \( t \) and \( r \) exchange their roles

at \( r = 0 \) metric has true singularity
Different types of black holes

• **Black holes formed in collapse of stellar matter**
  - endpoint of stellar evolution, mergers and accretion
  - mass range: a few $M_{\text{Sun}} - 10^9 M_{\text{Sun}}$

• **Primordial black holes**
  - early universe, large fluctuations in energy density
  - mass range: $M_{\text{Pl}} - M_{\text{Sun}}$

• **Black holes formed in the Lab**
  - need accelerator as big as the whole universe
  - mass range: a few $M_{\text{Pl}} \sim 10^{19} \text{GeV}$
Extra Dimensions

• Our world is manifestly (3+1)-dimensional on large scales

• Kaluza (1921) and Klein (1926) introduced the fifth dimension to unify gravity with electromagnetism

• 5th dimension rolled on a very small circle

• Size of extra dimension is \( L_{pl} \sim 10^{-33} \text{cm} \)
Higher dimensional objects?

• To unify all the interactions we need more than one extra dimension
  
  KK approach → at least 11 dimensions
  String theory → 10 or 11 dimensions

• We can have interesting higher dimensional objects
  e.g. higher dimensional black holes, topological defects (strings, branes etc)

• At distances of $L_{Pl} \sim 10^{-33}$ cm quantum gravity effects become very important

• Problem: quantum gravity has not been formulated yet →

  Can not describe them properly!
Brane worlds and large extra dimensions

- Brane world models have attracted a lot of attention
- Introduced as a solution to the hierarchy problem
- They imply existence of large extra dimensions
- They offer rich higher dimensional phenomenology
- We can study higher dimensional black holes!
The hierarchy problem

- Planck energy scale: \( M_{Pl} \sim 10^{19}\text{GeV} \gg M_{EW} \sim 200\text{GeV} \)
- "Grand Desert" between the scales
- \( G_{Newton} = 1/M_{Pl}^2 \rightarrow \text{weak gravity} \)
- Gravity is by far the weakest interaction in nature
- For protons, gravity is \( 10^{36} \) times weaker than electromagnetism

\[ F_{EM} = \frac{q_1 q_2}{r^2} \quad F_G = \frac{G m_1 m_2}{r^2} \]
The need for physics beyond the SM

• Validity of SM is probably limited to energies up to 1 TeV

• Radiative corrections to the Higgs mass:

\[ \Delta m_h^2 \approx \Lambda^2 \frac{3(2M_W^2 + M_Z^2 + m_h^2 - 4m_t^2)}{(32\pi^2 v^2)} \]

• If SM is valid all the way to \( M_{Pl} \), i.e. \( \Lambda \sim M_{Pl} \), then a rather fine-tuned cancellation must take place (about 1 part in \( 10^{17} \))

• If physics beyond the SM is to solve the hierarchy problem, it has to come not far above the TeV scale

• Supersymmetry

• Strong (TeV scale) quantum gravity
**Strong gravity --- ADD model**


- Our universe consists of:
  - 3+d space like dimensions (bulk)
  - d dimensions compactified to radius R

- **Only gravitons are allowed to propagate in all dimensions**

- **SM particles are bound to 3-dim submanifold (brane)**
In this framework:

- Gravity is as strong as the other interactions
- But gravitational force is diluted due to the presence of extra dimensions
- Weak gravity is only an illusion for an observer located on the brane
**Flat compact extra dimensions**

Volume of extra space: \( V_{extra} = R^d \)

\[
G_4 \equiv \frac{1}{M_{Pl}^2} = \frac{G_{4+d}}{V_{extra}}
\]

Fundamental scale \( M_* \sim 1\text{TeV} \)

Compactification radius: \( R \sim 10^{\frac{32}{d}} \cdot 10^{-19}m \)

\[
d = 1 \quad \Rightarrow \quad R \sim 10^{13}m \quad \text{(excluded)}
\]

\[
d = 2 \quad \Rightarrow \quad R \sim 1\text{mm} \quad \sim \text{current lab limit}
\]

\[
d = 3 \quad \Rightarrow \quad R \sim 10^{-5}\text{mm}
\]

\[
d = 6 \quad \Rightarrow \quad R \sim 10^{-11}\text{mm}
\]

``Large'' extra dimensions \( R \gg T\text{eV}^{-1} \)
Black Holes in accelerators

Particle accelerator (e.g. Large Hadron Collider):

Collision of two particles with COM energy $\sqrt{s}$

If an impact parameter $b$ is smaller than $2R_H$ for a given $\sqrt{s}$

$$b < 4G\sqrt{s}$$

Black hole with a mass $M = \sqrt{s}$ forms
Large Hadron Collider → CERN (2007)

$LHC: \sqrt{s} = 14\text{TeV}$

Geometrical cross section for black hole production:

$$\sigma(M) \approx \pi R_H^2$$

Numerical estimates:

$10^7$ black holes per year if $M_* = 1\text{TeV}$

$LHC - \text{black hole factory!}$
Hawking radiation

- Hawking (1973): black holes radiate

- Virtual particle-antiparticle pairs are being created all the time in vacuum

- Usually they disappear almost instantaneously

- If a virtual pair is created near the event horizon

  - one of the particles could escape and become real

- the black hole thereby loses its gravitational energy (mass)
• Black hole Hawking radiation is thermal!

• Black hole decays into all degrees of freedom available at a given temperature democratically

• Black hole Hawking radiation temperature: \( T_H = 1/R_H \)

• Number of particles emitted proportional to black hole entropy: \( S \sim \frac{R_H^2}{M_*^2} \)

• If \( M_* = 1\,\text{TeV} \) and \( N + 1 = 10 \) then \( M = 5\,\text{TeV} \) black hole emits of order 30 quanta

• **BH event quite different from any other SM event!**
Where do black holes mostly radiate? Brane or bulk?


“Black holes radiate mostly on the brane”

\[ \lambda_T > R_S \rightarrow \text{point radiator, s-mode dominant} \rightarrow \text{radiates equally in all directions} \]

# of deg. of freedom much larger on the brane? (60 SM particles vs 1 graviton)

# of deg. of freedom of gravitons in the N+1-dimensional space-time is

\[ \mathcal{N} = (N+1)(N-2)/2 , \text{ so for } N+1 = 10 \text{ we have } \mathcal{N} = 35! \]


• LHC: non-zero impact parameter \(\rightarrow\) rotating black holes

• Rotating black holes \(\rightarrow\) superradiance \(\rightarrow\) graviton emission dominant

**Black holes radiate mostly OFF the brane!**
Recoil Effect


• Any particle emitted in the bulk can cause a recoil of the black hole from the brane

• Recoil due to Hawking radiation can be very significant for small black holes (energy of emitted particles comparable to the mass of the black hole)

• Consequences:
  i) black hole radiation would be suddenly terminated
  ii) observer located on the brane would register apparent energy non-conservation
Friction between the black hole and the brane


Rate of loss of the angular momentum

\[ \dot{J} = \pi \sigma a r_H \cos^2 \alpha \]

\[ \alpha = \pi / 2 \quad \rightarrow \quad \dot{J} = 0 \]

final stationary equilibrium configuration
Black Holes from Cosmic Rays


• Cosmic rays are Nature’s free collider

• Observed events produce COM energy of 100 TeV

• If $M_* \sim$ TeV (quantum gravity energy scale), then

  — small black holes can be produced in the atmosphere —

• Proposed mechanism:

  - neutrino-nucleon scattering deep in the atmosphere
Cosmic rays

- What are cosmic rays?

- Energetic particles ($10^8$ eV - $10^{20}$ eV)

- Originate from space (solar, galactic, extragalactic origin)

- Primary cosmic rays: Hadrons, leptons, photons??
- Or more exotic “particles” – monopoles, heavy relics from Big Bang??

- Steep flux ($\sim 1/E^{2.7}$)
Pierre Auger Observatory

- The largest cosmic ray observatory
- Located in Argentina (Pampa Amarillas)
Pierre Auger Observatory

• 1600 Water Cerenkov ground arrays +
• 4 air fluorescence telescopes
• spread over 3000 km²

• Best setup for cosmic ray studies
Cosmic neutrinos

- Cosmic protons scatter off the cosmic microwave background to create ultra-high energy neutrinos
  \[ p + \gamma_{CMB} \rightarrow n + \pi^+ \rightarrow n + \mu^+ + \nu \]

- These neutrinos enter Earth's atmosphere

- They have very weak SM interactions

- Dominant interaction: \( \nu N \rightarrow BH + X \)
Crucial points:

• Neither strong nor electromagnetic interactions can degrade the neutrino energy before it interacts quantum-gravitationally.

• Neutrino interaction length is far longer than the thickness of the Earth's atmosphere.

• Neutrinos can produce black holes uniformly at all atmospheric depths.

• Protons and photons interact high in atmosphere and cause vertical showers.

• The most promising signal for neutrinos:
  - quasi-horizontal showers initiated by neutrinos deep in the atmosphere.
  - far above the standard model rate.
The total black hole production cross section in neutrino-nucleon scattering is:

\[ \sigma(\nu N \to BH) = \sum_i \int dx \, \hat{\sigma}_i(xs) f_i(x, \tilde{Q}) \]

- The sum runs over all partons in the nucleon

\[ s = 2m_N E_\nu, \quad E_{CM} = \sqrt{s} \]

- \( f_i \) are the parton distribution functions

- \( \tilde{Q} \) is momentum transfer

\[ \hat{\sigma} = \pi R_s^2 \]

- The cross section for black hole production is found to be several orders of magnitude higher than the SM cross section for \( \nu N \to LX \), if \( M_\ast \approx 1 - 10\text{TeV} \).
• Numerical estimates:

- Auger can detect $\sim 100$ black holes in 3 years
  (i.e. BEFORE the LHC data become available)

• This could be the first window into extra dimensions

• USA Today version:

"Dozens of tiny black holes may be forming right over our heads...
A new observatory might start spotting signs of the tiny terrors,
say physicists Feng and Shapere. They're harmless and pose no
threat to humans."
"Science may be described as the art of systematic over-simplification."

Karl Popper, The Observer, August 1982
Some things have their natural habitat in the "grand desert" that is destroyed by a low scale gravity.

- Like proton stability, neutrino masses...

- Low scale quantum gravity implies very fast proton decay!
Black hole mediated proton decay

- Small scales: space-time fluctuations large
- Hawking: these fluctuations are virtual black holes

\[ \tau_{proton} \sim m_{proton}^{-1} \left( \frac{M_{Pl}}{m_{proton}} \right)^4 \]

If \( M_{Pl} \sim 10^{19} \text{GeV} \), then \( \tau_{proton} \sim 10^{45} \) yrs

If \( M_{Pl} \sim 10^3 \text{GeV} \), then \( \tau_{proton} \sim 10^{-11} \) sec !!!
Gauging the baryon number

• One way out is to gauge the baryon number
  → promote a $U(1)_B$ into a gauge symmetry

• **Problems:**

• **Baryogenesis**
  - Before: "We exist → proton must be stable"
  - After:  "We exist → proton must be unstable"

• To avoid a new long range interaction, $U(1)_B$ must be broken down to some discrete gauge symmetry

• Arranging for anomaly cancellation

• Gauge couplings unification

• So far, gauging the baryon number has not proved very attractive!
An alternative - Split Fermions


• In order to suppress a direct QQQL coupling we must separate quarks form leptons

• Quarks and leptons are localized at different points on a thick brane
  • Or alternatively, on different branes

• The model yields exponentially small coupling (wave function overlap) between quarks and leptons

• In the language of virtual black holes:
  • Virtual black holes responsible for proton decay must be very large
    → have small probability to be created
• The propagator between fermions which are separated in extra dimensions (in the high energy and high momentum transfer limit) is

\[ P_{\text{extra}} \approx P_4 e^{-d^2/\sigma^2} \]

\( d \): separation between the quarks and leptons
\( \sigma \): the width of the fermion wave function

• The propagator has the usual 4-dim form except that the coupling is suppressed by the exponentially small wave function overlap

• Suppression factor of \( e^{-d^2/\sigma^2} \approx 10^{-26} \),
  (which can be achieved for a rather modest hierarchy of \( d \approx 10 \sigma \))

**completely saves the proton!**
Consequences: the price we have to pay


• Spatial separation between the quark and lepton wave functions successfully suppresses proton decay

• However, this implies strong consequences for cosmic ray neutrino scattering off the atmosphere

• The correct black hole production cross section in collisions of neutrinos with each quark in a nucleon is not \( \hat{\sigma} = \pi R_H^2 \)

• The correct cross section is multiplied by the large suppression factor of \( 10^{-52} \)
• Proton contains other partons besides quarks: e.g. gluons, other gauge bosons etc.

• **However:**

• Once you separate leptons from quarks, higher order processes are also highly suppressed

  • by exponential wave function suppression factors
  • by power law volume suppression factors

  …
Large suppression factors enter the total production cross section

\[ \sigma(\nu N \rightarrow BH) \]

and render the corresponding probability for the black hole production by cosmic neutrinos completely uninteresting for the Auger Observatory!
• Black holes might still be produced in NN or γN scatterings

• **Problems:**

• The Earth's atmosphere is not transparent to nucleons or photons as it is to neutrinos

• SM interactions much stronger

• One can not expect quasi-horizontal showers deep in the atmosphere

**No distinct experimental signature of BH production!**
What about lepton-lepton (e.g. $\nu e^-$) scattering?

- $m_e$ is 2000 times smaller than the $m_N$
- The threshold neutrino energy for leptonic black-hole production is 2000 times higher
- The cosmic neutrinos flux goes down steeply with energy ($\sim 1/E^{2.7}$)
- The expected suppression is $\approx 10^8$
- **Completely uninteresting for the Auger Observatory**
Conclusions

• Fine tuning in the SM implies either new physics or strong gravity at TeV scale

• If gravity is strong, we can expect non-perturbative quantum gravity effects at soon available energies in accelerators

• Like mini black hole production

**HOWEVER:**

• The weakest link in TeV scale gravity models → fast proton decay

• Realistic models with stable proton:
  Some of the channels for black hole production are strongly suppressed
Dialog between Lord Michael Faraday (1791 - 1867) and the Chancellor of the Exchequer

**QUOTATION:**

- Mr. Gladstone, then Chancellor of the Exchequer, had interrupted Lord Faraday in a description of his work on electricity to put the impatient inquiry:

  “Very well Lord Faraday, but after all, what is the use of it?”

  "Well sir, there is every probability that you will soon be able to tax it!"