Basic SM processes at the Large Hadron Collider

Known and important unknown QCD effects

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New phenomena at TeV energies

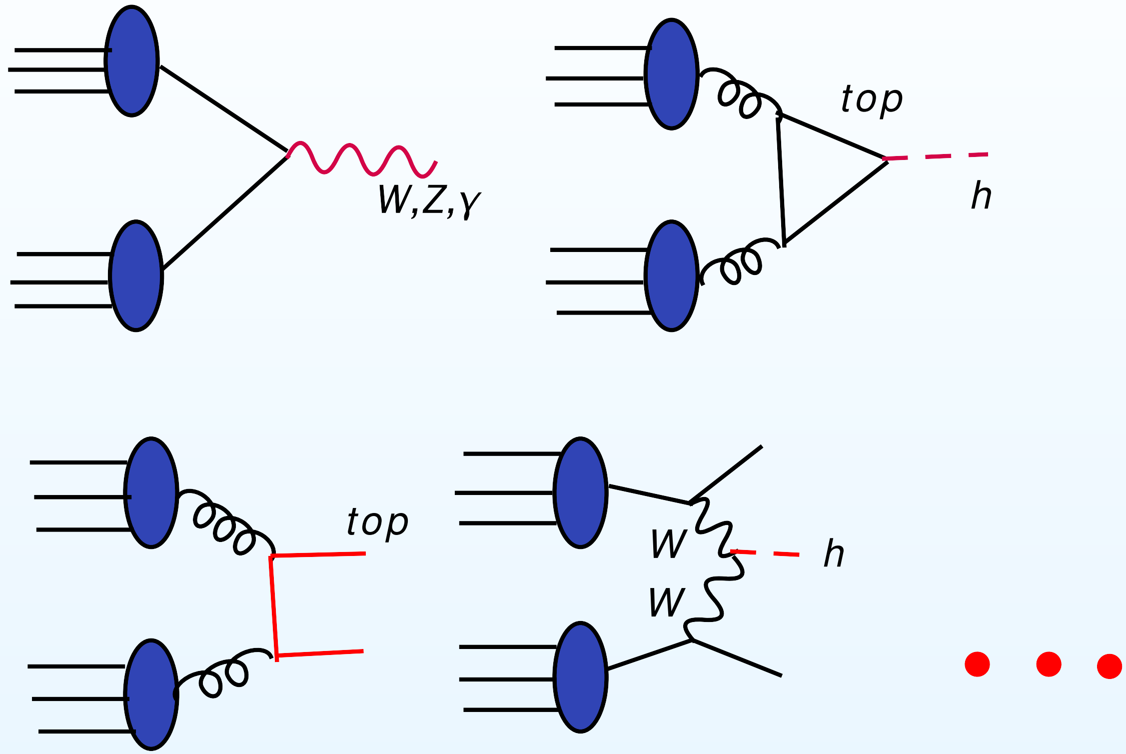
- LEP, SLC, Tevatron, muon experiments, B-meson factories, $\ldots$
  - *Discovered all Standard Model particles except the Higgs boson!*
  - *Few inconclusive ($< 3\sigma$) deviations:* $(g-2)_\mu, \sin^2\theta_w, \ldots$
  - *A host of precision measurements, pointing to a light Higgs boson!*
- The SM is not the whole story:
  - *Massive neutrinos.*
  - *Dark matter + dark energy.*
  - *Gravity?*
- Revolutionary theoretical possibilities:
  - *Supersymmetry, Extra dimensions.*

The LHC will give us a unique opportunity for new discoveries.

Large Hadron Collider

- Starting operation in 2007
- Collisions of 7 TeV proton beams
- Luminosity 10 - 100 fb^{-1}/year
  - Small statistical uncertainties $1\% - 2\%$ will be easily achieved.
  - Very good detectors.
- High rates could allow both discoveries and many descent precision studies.

Processes at the LHC

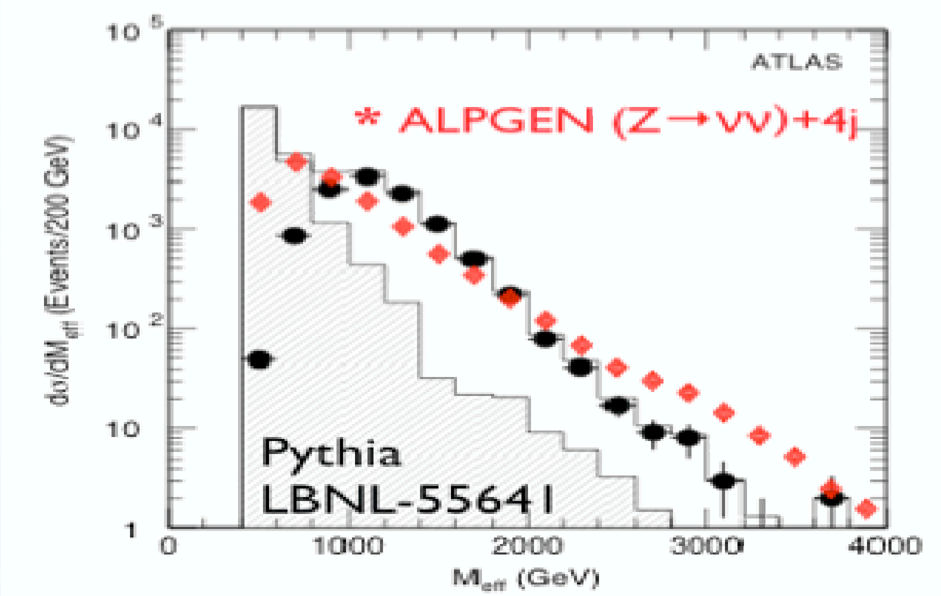


- A vast experimental program: pointless to give a list of "interesting" processes.
- We hope to discover many new BSM processes. But even within the SM, there is a lot to do!

In this talk

- Basic cross-sections will be measured very well.
  - $pp \to W, Z$
  - $pp \to h$
  - $pp \to jets$
  - $pp \to W, Z +$ jets
  - $pp \to W^+W^-, Z^+Z^-$
  - $pp \to t\bar{t}$
- How good are the current theoretical computations?
- Innovation in perturbative methods. What can we hope for?

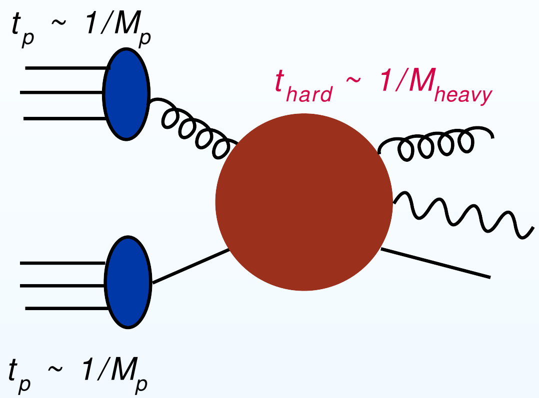
How wrong can we get it?



Mangano

- Supersymmetry signal: Jets + missing energy.
- Two approaches to describe the background.
  - Shower Monte-Carlo
  - Exact matrix-elements at leading order in α_s
- Two different aproximations: we should learn how to use them, and their limitations.

Hard scattering processes in hadronic collisions



- Jets with high transverse momentum to the beam axis and heavy particles are produced very fast:

$$\tau_{hard} \sim \frac{1}{m_{heavy}}$$

- Interactions inside the protons are too slow

$$\tau_{protons} \sim \frac{1}{M_{proton}} >> \tau_{hard}$$

to change the proton content during the hard scattering.

Factorization theorem

- The protons are beams of quarks and gluons $p_1 = x_1 P_1, p_2 = x_2 P_2$.
- $f_i(x)$: Momentum distributions of quarks and gluons in the proton do not change during the scattering.

$$\sigma = \sum_{ij} \int f_i^{proton}(x_1)\, f_j^{proton}(x_2)\, \sigma_{ij}(x_1, x_2)$$

- Cross-section for the hard scattering of quarks and gluons factorizes. At large scales the strong coupling is small; we can use perturbation theory

$$\sigma_{ij} = a_s^N \left(\text{Leading Order} + \alpha_s \text{ Next LO} + \alpha_s^2 \text{ NNLO } + \ldots\right)$$

Does this really hold beyond the LO? YES

Collins, Soper, Sterman, . . .

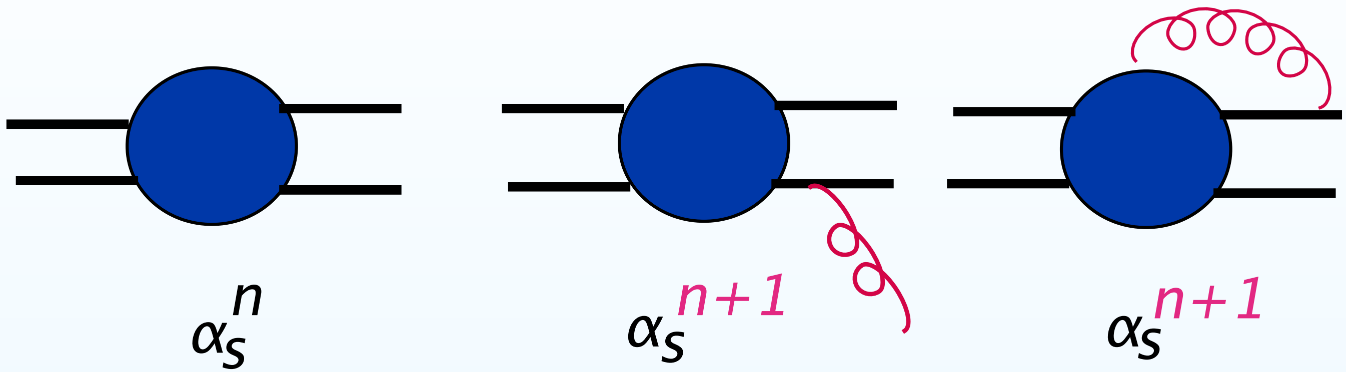
- Perturbation theory includes all types of interactions:

$$\int \frac{d^4k}{k^2 \ldots}$$

Infrared: $k \sim 0$, *hard:* $k \sim M$, and *ultraviolet:* $k \sim \infty$!

- The contributions of the ultraviolet interactions renormalize the physical values of the Lagrangian parameters, e.g. α_s.
- Some infrared interactions do not contribute (cancelations) in interesting physical observables.
- The remaining infrared interactions change the parton distribution functions.

What is the expansion parameter?



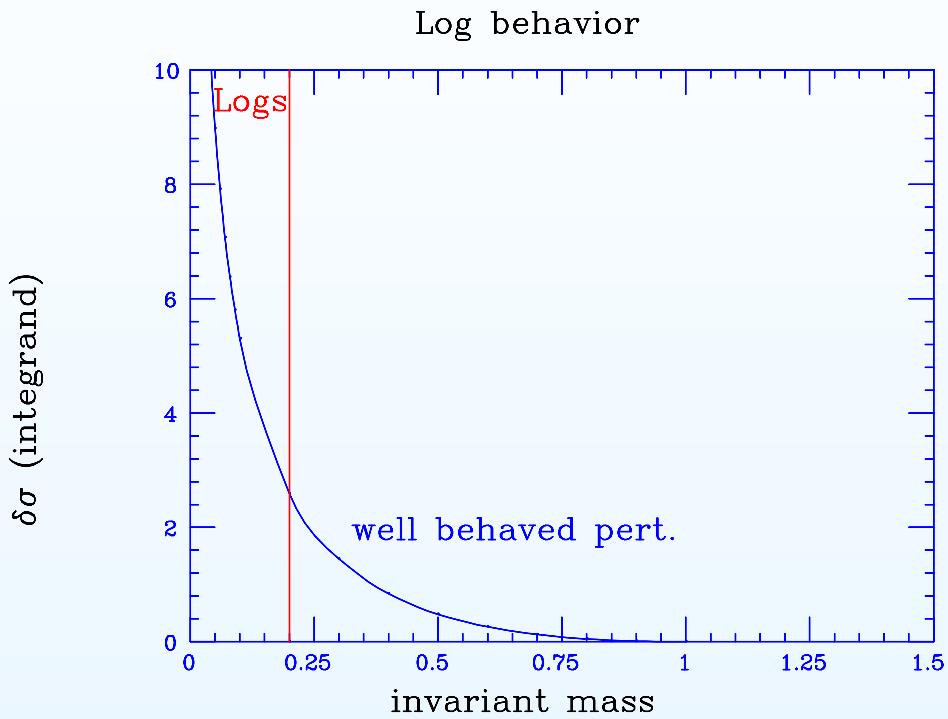
- Virtual emission+real emission:

$$\alpha_s^{n+1}\left(-\frac{A_2}{\epsilon^2}-\frac{A1}{\epsilon}+A_0\right)$$

$$+\alpha_s^{n+1}\left(\frac{A_2}{\epsilon^2}-\frac{A1}{\epsilon}+\ln\left((p_i+p_g)^2\right)\ln\left((p_j+p_g)^2\right)+\ldots\right)$$

New logs emerge at each order in α_s. These are large in the phase-space regions where radiation is soft ($E \longrightarrow 0$) or collinear $\vec{p_g} \sim \lambda \vec{p_i}$.

Logs in fixed order perturbation theory



- The perturbative corrections could be large if our observables probe a lot the IR regime (e.g. with jet-vetos).
- Fixed order perturbative expansions will give wrong results
- Resummation or finding alternative observables are then needed.

Parton-shower Monte-Carlos

- Works when logs are dominant.
- Resum leading logs from multiple independent infrared emissions probabilisticaly

$$\sum_n \alpha_s^n \ln^{2n}(s_{ig}) \rightsquigarrow s_{ig}^{\alpha_s}$$

- This is possible since such emissions factorize. Subleading logs and hard emission contributions are dropped.
- Simple method to obtain rough estimates; Feynman diagrams with many external particles are usually intractable
- Main simulation tool at colliders (HERWIG, PYTHIA, . . .).

When can parton showers seriously fail?

- If we measure events away from the log dominant phase-space regions (soft/collinear)
- Such (high p_T) configurations are the most interesting at the LHC
- Heavy new particles will decay into many jets and leptons with high p_t.
- This type radiation is characteristically different than soft and collinear radiation in QCD background processes.
- “New physics” experimental cuts (selecting high p_t) require the study of background processes wih matrix-element calculations; not (only) with parton showers!

Z, W + N jets

- Produce signatures for leptons + many jets and missing energy

$$Z \to \mu^+\mu^-, e^+e^-, \tau\bar{\tau}, \nu\bar{\nu}, jets$$

$$W \to l\nu, jets$$

- How well can we compute theoretically the backgrounds e.g. $Z + 3, 4, 5 jets$?
- Currently, we can answer this question at leading order in perturbation theory (e.g. ALPGEN)

Leading order prediction

Leading order scale variation for $pp \to \nu\bar{\nu} + N$jets Select high $p_t > 80\,\mathrm{GeV}$, central $|\eta| < 2.5$ jets. Pick a reasonable scale:

$$\mu^2 = M_Z^2 + \sum_{jet} p_{t,jet}^2$$

and allow to vary: $\mu_R = \mu_F = \mu/2 - 2\mu$

N	$\sigma(2\mu)[pb]$	$\sigma(\mu/2)[pb]$	variation
1	182	216	17%
2	47.1	75.4	46%
3	6.47	13.52	70%
4	0.90	2.48	93%

ALPGEN

For a 5σ discovery with LO magnitudes: $\rightsquigarrow$ Signal > 2.5 Background

NLO predictions?

- So far, only $pp \to Z + 1, 2$ jets is known at NLO MCFM
- The scale variation is less than 15%.
- Challenge: construct NLO Monte-Carlo programs for $pp \to Z + 3, 4, 5$ jets!
- If we succeed we will have trully quantitative predictions for these cross-sections at the level of $\sim 30\%$!
- NLO computations for high multiplicities are tough!!! Can we live without?
- A goal of the experimental analysis should be to reduce as much as possible theoretical input (Monte-Carlos)

An experimenter's favorite: measure and extrapolate

- Measure $Z + N jets$ “inclusive” ratios from $1 fb^{-1}$, where the new physics contribution should be small

$$\frac{\sigma^3_{incl}}{\sigma^2_{incl}}, \frac{\sigma^4_{incl}}{\sigma^2_{incl}}, \frac{\sigma^5_{incl}}{\sigma^2_{incl}}$$

- Estimate number of high Z $p_t > 200\mathrm{GeV}$ events from:

$$\sigma^{3,4,5}_{p_t>200} = \sigma^2_{p_t>200} \frac{\sigma^{3,4,5}_{incl}}{\sigma^2_{incl}}$$

  , assuming that $\sigma^2_{p_t>200}$ is free of sizable new-physics contributions.

- Normalize Monte-Carlo's to $\sigma^{3,4,5}_{p_t>200}$. Use the tuned Monte-Carlo's to compute the cross-sections with the SUSY cuts. Anticipated precision in CMS TDR: 5%!!

What must be studied?

- Is this ratio sensitive to the p_t cut on Z at LO and NLO?

$$\frac{\sigma^{3,4,5}_{p_t>200}}{\sigma^{2}_{p_t>200}} = \frac{\sigma^{3,4,5}_{incl}}{\sigma^{2}_{incl}} ?$$

- Can we tune Monte-Carlo's to reproduce the correct distributions? There is no chance that parton showers on their own (Pythia, Herwig,) can describe this high p_t physics.
- We must work with Monte-Carlo's which switch from a parton-shower to a LO matrix-element description when away from the soft and collinear regions (Sherpa, Alpgen, $\ldots$).
- Succesful at the Tevatron! Will this be succesful at the LHC?
- Validation with NLO computations is indispensable!!!

Available next to leading order calculations

- Numerous calculations have been made at NLO.
- Anything you can think of for $2 \to 2$ processes: $pp \to WW$, $pp \to \gamma\gamma$, $pp \to t\bar{t}$, $\ldots$
- Many but not all, $2 \to 3$ processes: $pp \to \leq 3$jets, $pp \to W, Z+ \leq 2$jets, $pp \to qqh$, $pp \to tth$, $\ldots$
- No $2 \to 4$ process for the LHC. Only example of close enough complexity $e^+e^- \to 4$ fermions , Denner, Dittmaer, et al.
- What is the problem? Gigabyte sized expressions!

New attempts to solve the problem

- Clever methods to express SM amplitudes to the integrals which appear in a scalar ϕ^3 and ϕ^4 theory
  - Modified reductions to compactify expressions, avoid fake singularities, . . ., Denner, Dittmaer
  - Numerical reduction to master integrals, Ellis, Giele, Zanderighi; Giele, Glover
- Direct numerical integration
  - Subtraction method for loop amplitudes; numerical evaluation of Feynman parameter integrals, Nagy, Soper
  - Numerical evaluation of complex amplitude representations, CA, Daleo
- Reconstructing loop amplitudes from trees
  - Improved unitarity method I Bern, Berger, Dixon, Forde, Kosower Xiao, Yang, Zhu; Binoth, Gulliet, Heinrich
  - Improved unitarity method II Britto, Cachazo, Feng, CA, Mastrolia,Kunszt

Recent theoretical breakthrough

del Aguila, Pittau; Ossola, Papadopoulos, Pittau

- Discovered a miraculous functional form for generic loop integrands!

$$Amplitude = \int d^d k \left(A_1 \frac{1}{Den_1 Den_2 \ldots Den_n} + B_1 \frac{Spurious_1(k)}{Den_1 Den_2 \ldots Den_n} \right.$$
$$\left. + \sim 60 \text{ more terms} \right)$$

- first term (A_1) inegrates to a known scalar integral; spurious term (B_1) integrates to zero
- A_1, B_1 are found by evaluating the INTEGRAND at a sufficient number of values for the loop momentum where the propagators are on-shell.
- NLO calculations are theoretically of similar complexity as in evaluating many LO high multiplicity cross-sections.

NLO wishlist; what can be done?

- Explicit NLO calculations will help us understand how well we understand backgrounds at the LHC.
- Goals have been set very clearly for what must be computed:

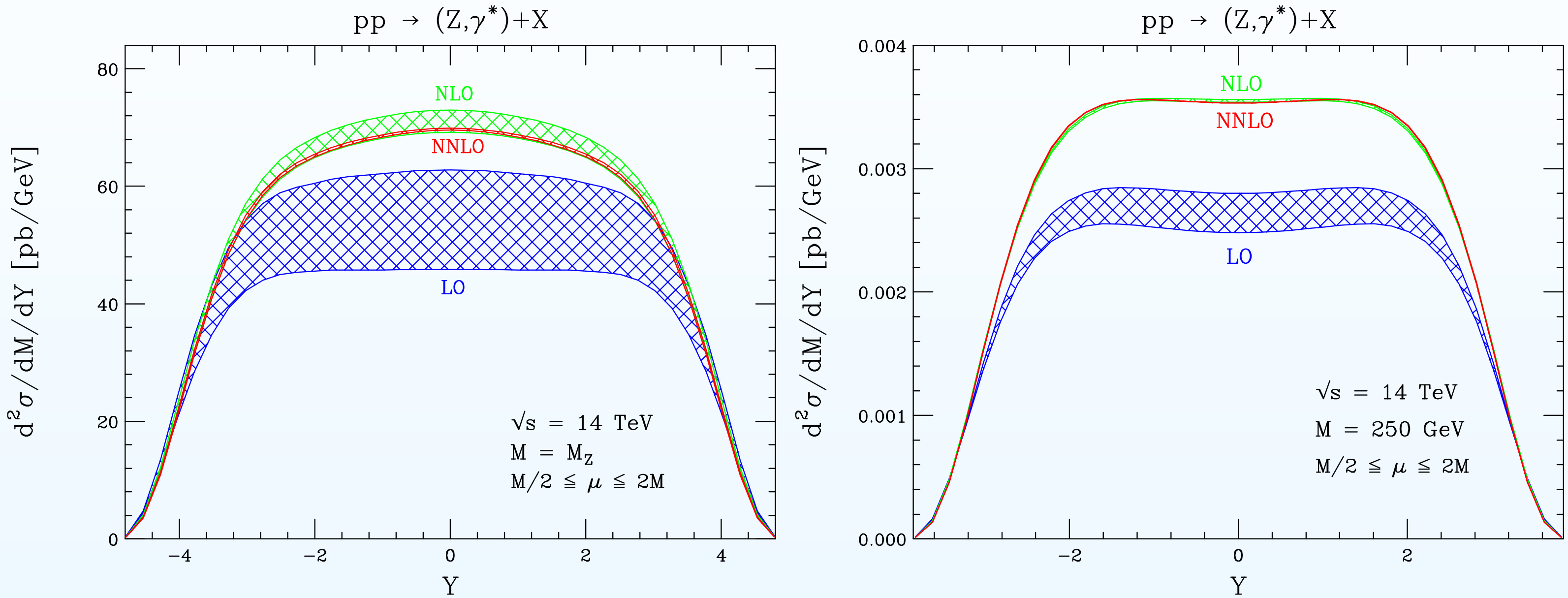
$$pp \to VV + 1, 2\text{jets}, t\bar{t} + 1, 2\text{jets}, VVV, V + 3, 4\text{jets}$$

- A very complex field theory + computational problem.
- Not all results will arrive as soon as we have hoped for, i.e. before the start of the LHC.
- We have just started to learn how to make computer implementations. Efficiency, will be a major issue.
- Computational resources (cluster/Grid computing) will be vital.

Phenomenology at NNLO?

- It is often said that:
  - LO rates can be used as order of magnitude estimates
  - NLO rates are reliable quantitatively
  - NNLO should be very precise
  - Alternative opinion: “NNLO should be too small to compute”
- There are two LHC processes which have been computed at NNLO: Drell-Yan, and Higgs production.
- What do we learn from these NNLO studies?

Drell-Yan lepton-pair rapidity through NNLO



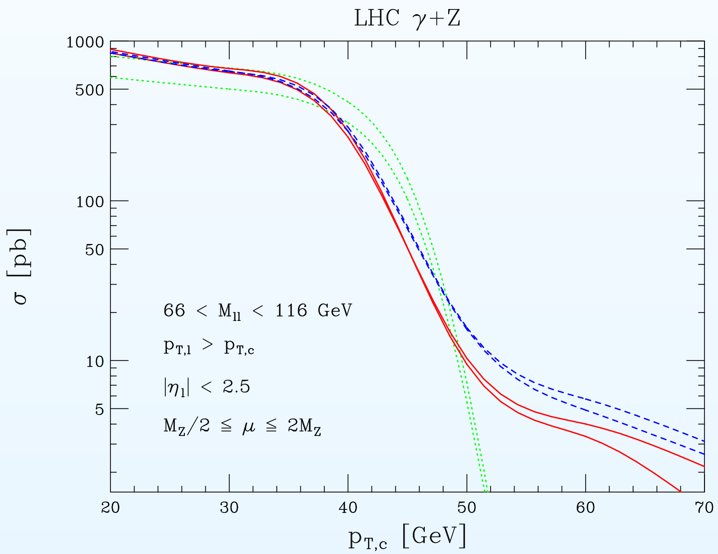
CA,Dixon,Melnikov,Petriello

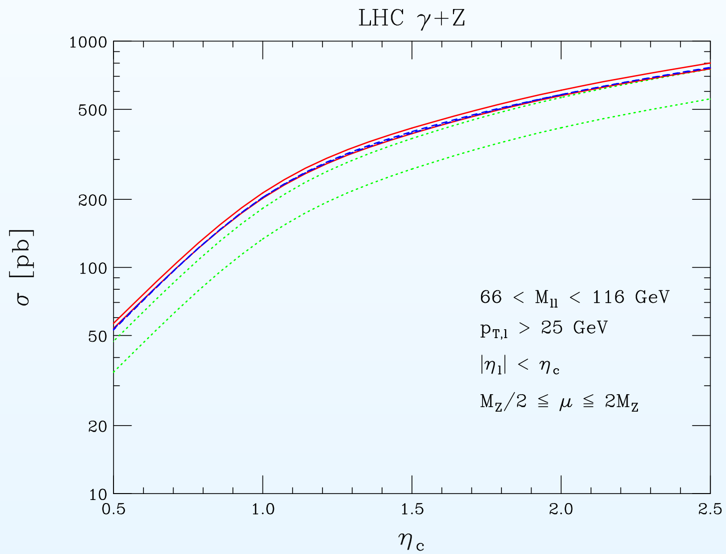
- NNLO validates the NLO result, and improve the scale variation uncertainty from 5% down to 1%.
    - Luminosity measurement
    - M_w, $\sin^2\theta_w$, extraction
    - Constrain parton densities

Fully differential NNLO Monte-Carlo for Drell-Yan

Melnikov, Petriello

- Contains spin correlations, finite-width effects, $\gamma - Z$ interference, all kinematics

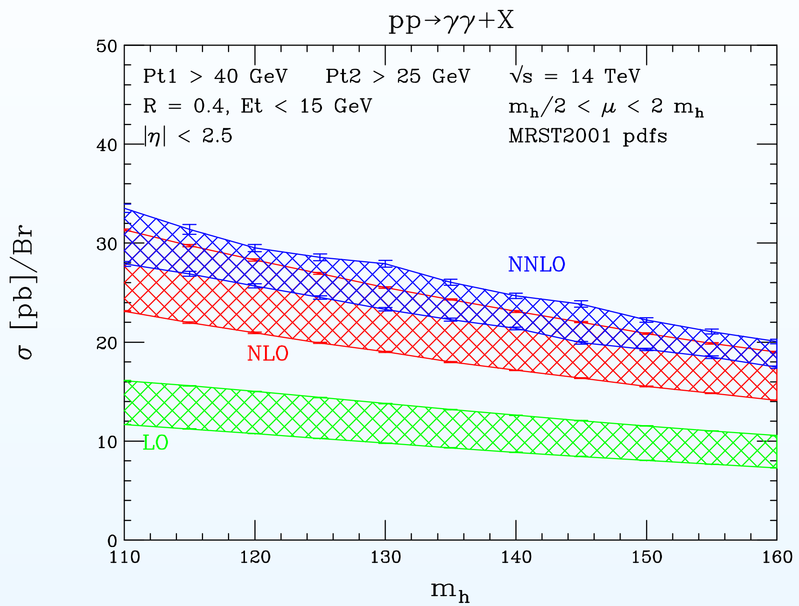




Melnikov,Petriello

- Amazingly precise NNLO predictions.
- Direct comparison with experiment; all cuts at the parton level are included!

Di-photon signal cross-section in Higgs production



CA, Melnikov, Petriello

$$\mathrm{NLO} \simeq 1.7\mathrm{LO}$$

$$\mathrm{NNLO} \simeq 2\mathrm{LO}$$

- DY@NNLO: improves error estimate; confirms NLO as a realistic estimate of the cross-section
- Higgs@NNLO: necessary to obtain a reliable estimate

$pp \to t\bar{t}$

- The LHC will be a top-quark factory.
- Many extensions of the SM modify the top-quark sector (large top mass)
- Background to many new physics searches
- Anticipated precision (CMS TDR) at the LHC $(10 fb^{-1})$:

channel	Syst.$(\%)$	Stat.$(\%)$	Lum.
dilept. $(10 fb^{-1})$	11	0.9	3
tau-lept. $(10 fb^{-1})$	16	1.3	3
semi-lept. $(10 fb^{-1})$	9.7	0.4	3
hadr. $(1 fb^{-1})$	20	3	5

The $pp \to t\bar{t}$ K-factor

Campbell, Huston, Stirling

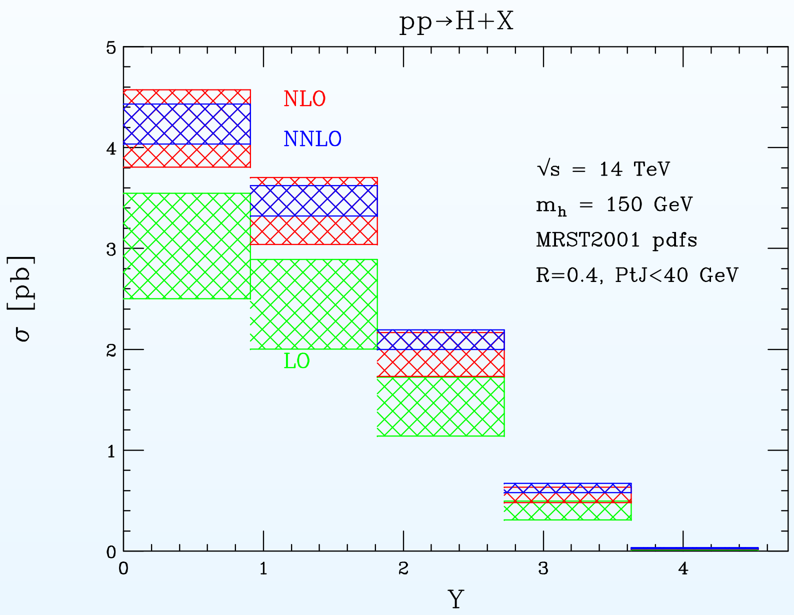
- The NLO K-factor is:

$$\frac{\text{NLO}}{\text{LO}}(@Tevatron) = 1.24 \quad \frac{\text{NLO}}{\text{LO}}(@LHC) = 1.48$$

- The K-factor at Tevatron depends strongly on M_{tt} while it is insensitive at the LHC.
- NLO components:
  - tree and one-loop for $pp \to tt + 0\text{“}jets''$ $\leadsto$
  - only tree for $pp \to tt + 1\text{“}jets''$ $\leadsto$ LO estimate
- At the LHC, $\frac{\Delta\sigma_{1-jet}(p_t > 30GeV)}{\sigma_{NLO}} \sim 60\%$. Smal x-gluons tend to produce significant additional initial state radiation
- A future NNLO calculation will stabilize such configurations. We will also learn a lot by a full NLO $pp \to t\bar{t}j$ calculation.

Jet-veto to improve convergence/ scale variation

- It works for (small-x gluon initiated) Higgs production. We might(!) find something similar for the NNLO $t\bar{t}$ cross-section.



CA, Melnikov, Petriello

- Be careful with choosing a jet-veto value where fixed order perturbation theory is still valid.
- Resummation is important (Catani, de Florian, Grazzini; . . .).
- Need for more flexible observables in resummation (Banfi, Salam, Zanderighi)
- Be careful with jet-vetos in the “intermediate” p_t region: sensitive to matching.

What is available at NNLO

- Drell-Yan total cross-section Matsuura, Hamberg, van Neerven (1991) Harlander, Kilgore (2002)
- Higgs boson (h,A) total cross-section Harlander, Kilgore (2002) CA, Melnikov (2002) Ravindran, Smith, van Neerven (2003)
- Drell-Yan rapidity distribution CA, Dixon, Melnikov, Petriello (2003)
- Splitting functions Moch, Vogt, Vermaseren (2004)
- Higgs boson fully differential cross-section CA, Melnikov, Petriello (2004)
- W-boson fully differential cross-section Melnikov, Petriello
- Two-loop amplitudes (but not yet the cross-sections) for $pp \to 1jet + X$, $pp \to \gamma\gamma$, $pp \to \gamma jet$, $pp \to W, Z + 1jet$, $pp \to h + 1jet$ CA, Glover, Oleari, Tejeda-Yeomans; Bern, Dixon, De Freitas, Ghinculov; Garland, Glover, Gehrmann, Koukoutsakis, Remiddi

Technical challenges at NNLO

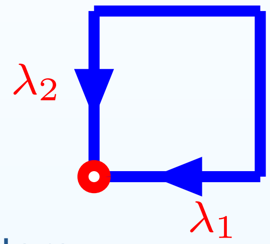
- Reduce two-loop amplitudes to master integrals (roughly, the integrals in scalar ϕ^3 and ϕ^4 theories) (!)
- Compute master integrals (!!)
- Cancel infrared divergences between real and virtual emission (!!!!)
    - Singular infrared emission at NLO from one gluon is easily parameterized in terms of two factorizable phase-space variables.
    - Singular NNLO emission from two gluons is described in terms of five non-factorizable variables.
- Write stable numerical code: slow Monte-Carlo adaptation to the multitude of peaks in the cross-section (!!!).

New (decade) techniques for NNLO computations

- Automated reduction to master integrals ⌣ Laporta; Gehrmann, Remiddi; CA, Lazopoulos, . . .
- Evaluation of master integrals
  - Novel differential equations ⌣ Kotikov; Gehrmann, Remiddi
  - Analytic and numerical Mellin-Barnes ⌣ Smirnov; Tausk; C.A., Daleo; Czakon
  - Numerical sector decomposition ⌣ Binoth, Heinrich; CA, Melnikov, Petriello
- Cancelation of infrared divergences
  - Generalized NLO-like subtraction Weinzierl; Kosower; Gehrmann-de Ridder, Gehrmann, Glover, Heinrich; Kilgore; Frixione, Grazzini; Somogyi, Trocsanyi, del Duca
  - Sector decomposition ⌣ CA, Melnikov, Petriello; Binoth, Heinrich

Extracting singularities with sector decomposition

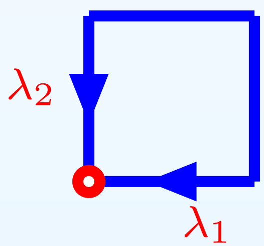
- Singularity when two (or more) variables reach the same corner

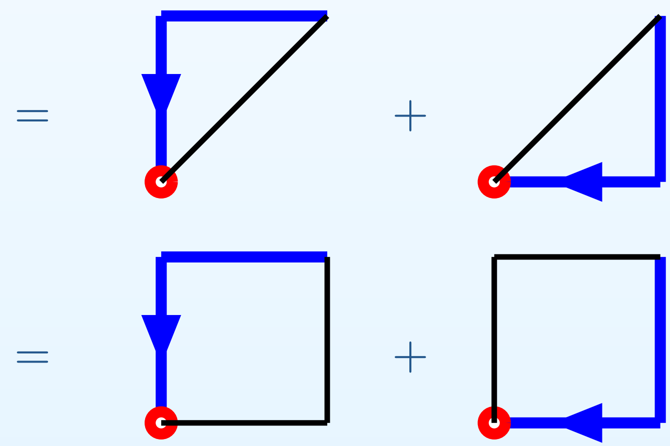


$$: \quad \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1+\lambda_2)^2} f(\lambda_1,\lambda_2;Obs(\lambda_1,\lambda_2))$$

- Split into sectors

Binoth, Heinrich; Denner, Roth; Hepp





- map each sector to $[0,1]$

- Repeat until singularities are fully factorized in all phase-space variables.

Wishlist fo NNLO

- NNLO computations could bring theoretical uncertainties bellow experimental systematic errors.
- My prioritized wishlist:

$$pp \to t\bar{t}, \quad pp \to V\bar{V}, \quad pp \to jet{+}X, \quad pp \to jet\gamma, \quad pp \to jetZ$$

  - Consolidate predictions for cross-sections with large or peculiar perturbative behavior.
  - Extract gluon density reliably
- No NNLO calculation for a $2 \to 2$ process so far. Unclear whether the methods under development and the methods for Drell-Yan and Higgs production will meet the challenge.
- Large overlap with developing NLO methods for many legs.

Higher order corrections and formal applications

- There has been an influx of ideas for QCD applications from string theory and more formal fields (Nigel Glover's talk in 2 weeks)
- But we have also seen the opposite. New methods (analytic and numerical evaluation of multi-loop integrals) and insight from QCD calculations (infrared structure of two-loop QCD amplitudes and resummation) has driven amazing developments in formal fields.
  - Factorization and exponentiation of amplitudes in $N = 4$ supersymmetric Yang-Mills - Integrability
  - AdS/CFT correspondence
- Higher order calculations are useful for phenomenology
- Higher order calculations deepen our understanding of field theory
- A very modern branch of particle physics combining powerful mathematics and computing.

Conclusions

- The LHC will require a systematic study of higher order QCD effects in basic processes.
- A lot of progress has been achieved in recent years.
- Further technical development is necessary.
- Hopefully, we will be able to make confident statements about the new physics at TeV energies trusting our “homework” for SM processes!