
Twistor inspired developments in perturbative QCD

Nigel Glover

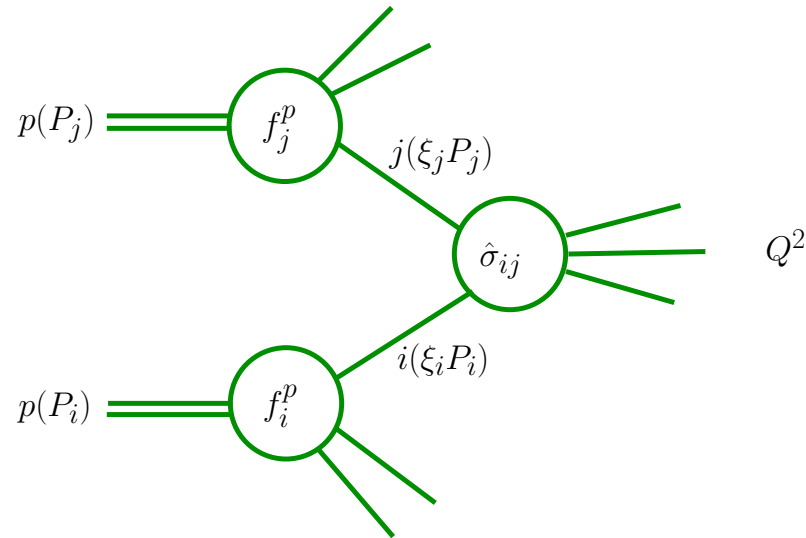
IPPP, University of Durham



Oxford University

20 February 2007

Hard processes in Hadron-Hadron collisions

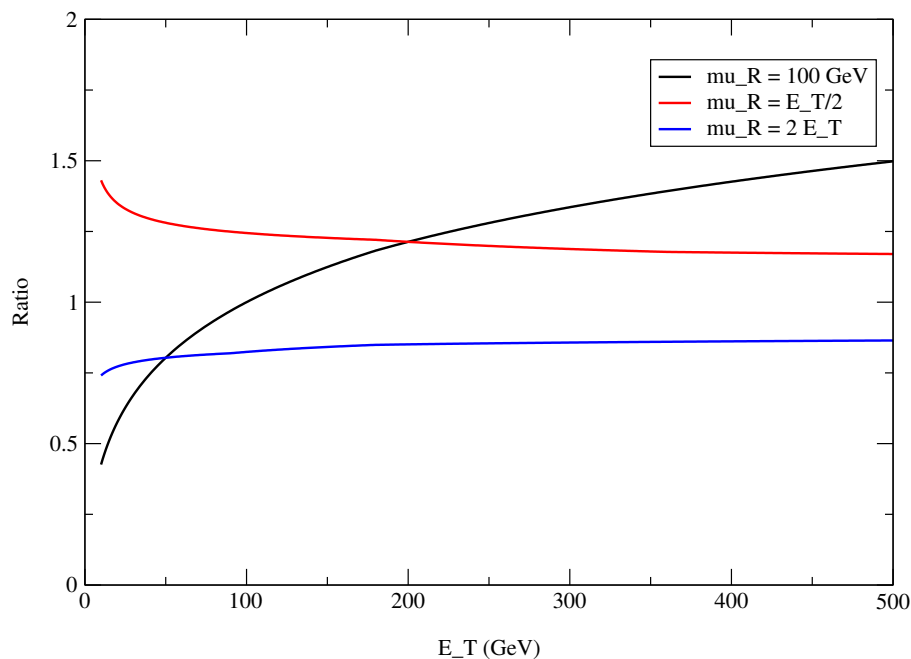


$$\sigma(Q^2) = \sum_{i,j} [\hat{\sigma}_{ij}(\alpha_s(\mu^2), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F^2) \otimes f_j^p(\mu_F^2)]$$

- partonic cross sections $\hat{\sigma}_{ij}$
- parton distributions f_i
- renormalization/factorization scale μ_R/μ_F
- + parton shower + hadronisation model

The unphysical scales - μ_R

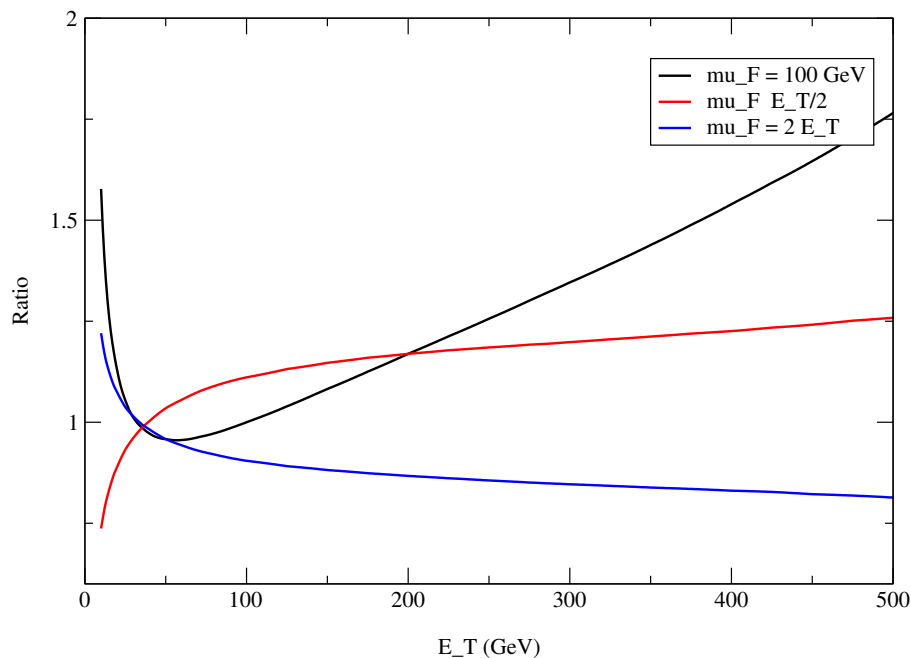
The **renormalisation scale** μ_R is introduced when redefining the bare fields in terms of the physical fields at scale μ_R . It is unphysical - and the answer shouldn't depend on it - but does because we work at a fixed order in perturbation theory. Therefore, you can choose any value (within reason). **Typical** values are the hard scale in the process $\mu_R \sim E_T$.



Example: $pp \rightarrow \text{jet} + X$ at **LO**
 α_s^2 for various values of μ_R compared
to $\mu_R = E_T$

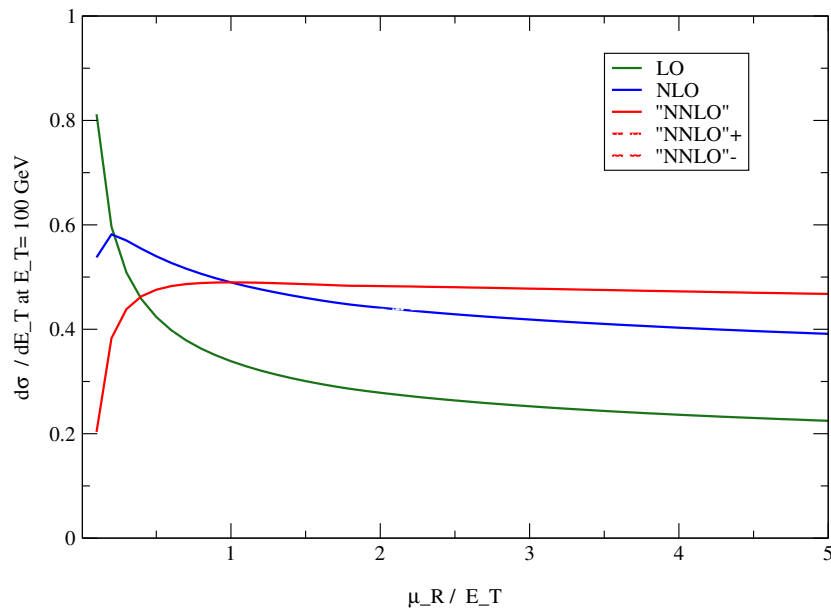
The unphysical scales - μ_F

The **factorisation scale** μ_F is introduced when absorbing the divergence from collinear radiation into the parton densities. It is unphysical - and the answer shouldn't depend on it - but does because we work at a fixed order in perturbation theory. **Typically** we think of radiation at a transverse energy $> \mu_F$ as being detectable so that $\mu_F \sim E_T$ is a reasonable choice.



Example: $pp \rightarrow \text{jet} + X$ at **LO**
The effective parton-parton luminosities for various values of μ_F compared to $\mu_F = E_T$ at $\eta_1 = \eta_2 = 0$

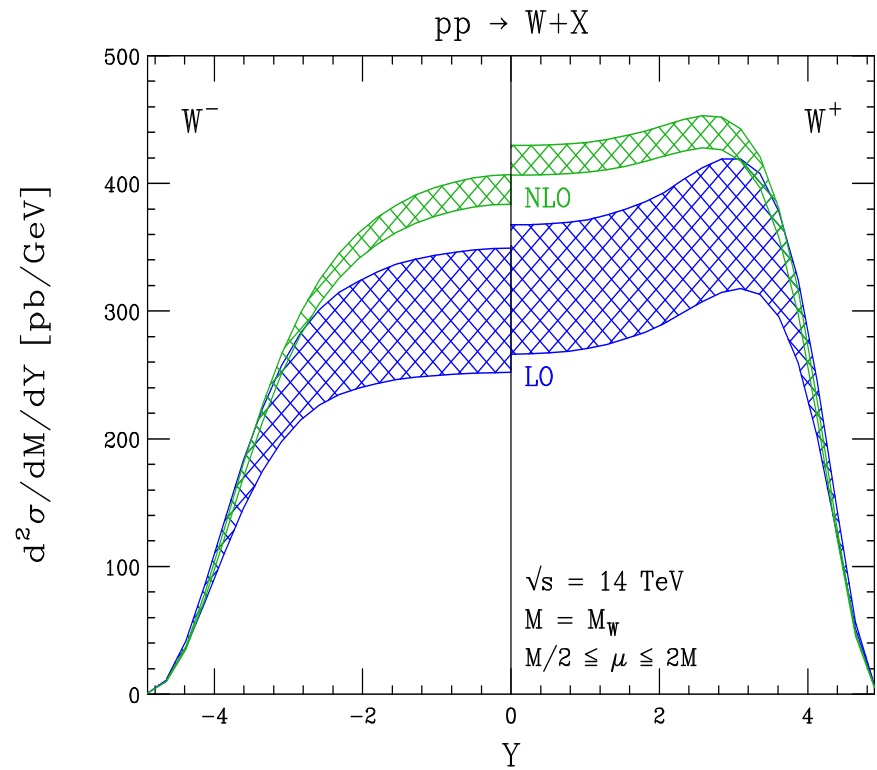
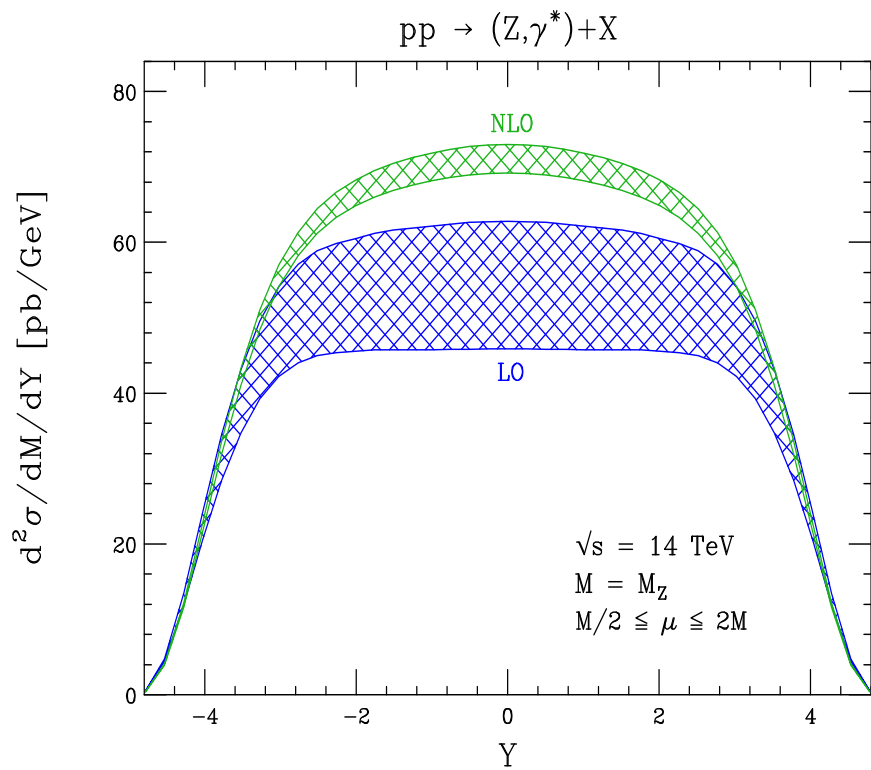
Unphysical scale dependence



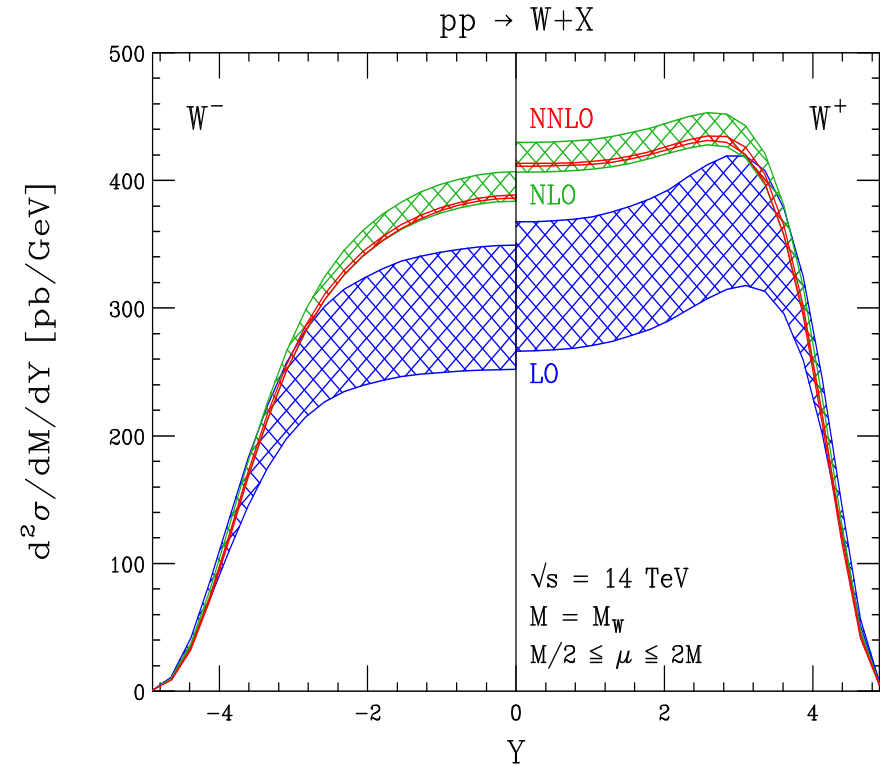
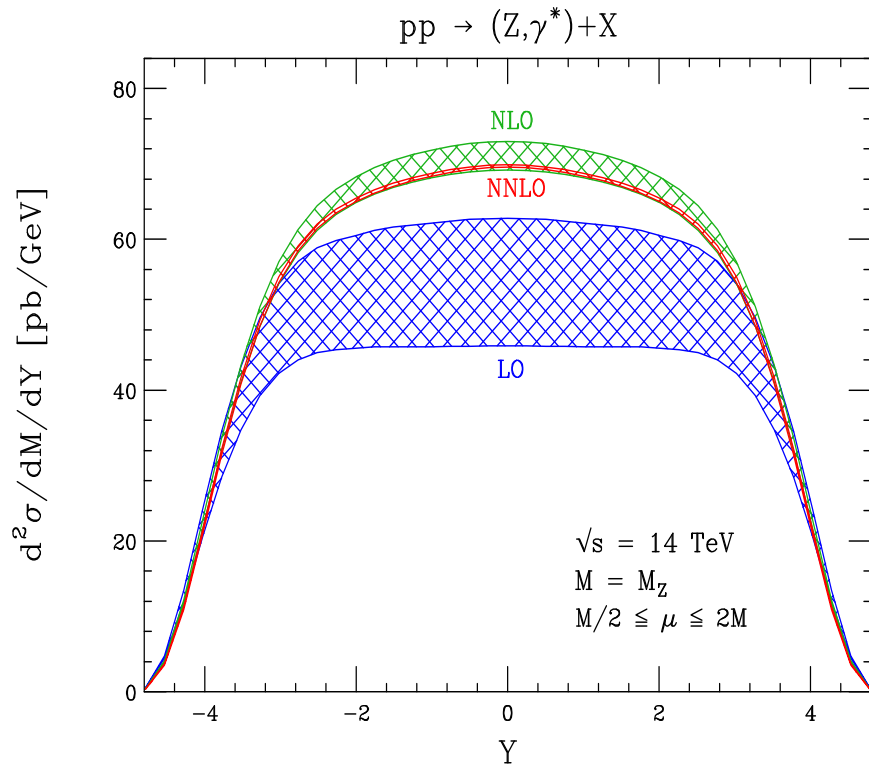
- typically, **NLO** reduces scale uncertainty by factor 2 over **LO**
- ✓ maybe to $\pm 30\%$
- typically, **NNLO** reduces scale uncertainty by factor 2 over **NLO**
- ✓ maybe $\pm 10\%$
- ✗ won't know till you do it

✓ plus many other improvements in modelling hard scattering at **NNLO**

Gauge boson production at the LHC



Gauge boson production at the LHC



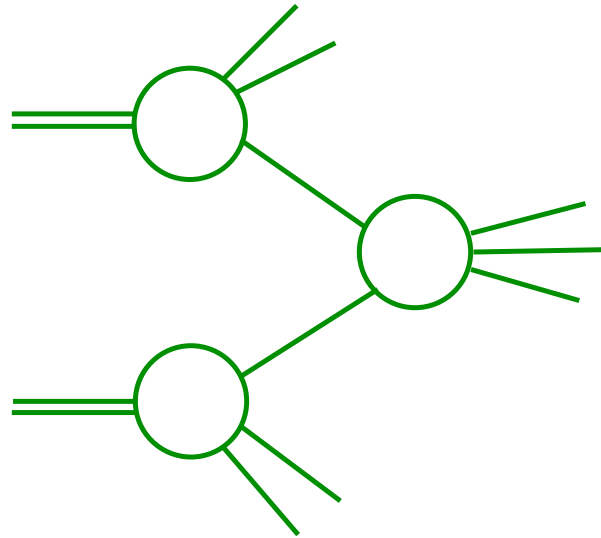
Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello

At LHC NNLO perturbative accuracy better than 1%

⇒ use to determine parton-parton luminosities at the LHC

Collider Physics



Concentrate on multiparticle final states that occur at high rate and form background to **New Physics**

High multiplicity, but low order - typically **LO** or **NLO**

For example, $pp \rightarrow V + 4 \text{ jets}$ is background to $pp \rightarrow t\bar{t}$ and other new physics.

State of the Art

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- LO ✓ matrix elements automatically generated up to $2 \rightarrow 8$ or more
- ✓ plus automatic integration over phase space
HELAC/PHEGAS, MADGRAPH/MADEVENT, SHERPA/AMEGIC++, COMPHEP, GRACE, ...
 - ✓ able to interface with parton showers - CKKW
 - ✓ very good for estimating importance of various processes in different models - properly populate phase space with multiple hard objects
 - ✗ rate very dependent on choice of renormalisation/factorisation scales

State of the Art

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1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- NLO** ✓ parton level **integrators** available for most $2 \rightarrow 2$ Standard Model and MSSM processes for some time
- ✓ extensively used at LEP, TEVATRON and HERA
EVENT, JETRAD, MCFM, DISENT, etc
 - ✓ **reduced** renormalisation scale uncertainty
 - ✓ can be matched with parton shower **MC@NLO** – **Frixione, Webber**

State of the Art

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- NLO ✓ some $2 \rightarrow 3$ processes available at NLO
 e.g. backgrounds $pp \rightarrow 3 \text{ jets}, V + 2 \text{ jets}, \gamma\gamma + \text{jet}, V + b\bar{b}$
 as well as signals $pp \rightarrow t\bar{t}H, b\bar{b}H, qqH, HHH, t\bar{t} + \text{jet}$
- ✗ many still missing $VV + \text{jet}, t\bar{t} + \text{jet}$, etc
- ✗ understood how to do, but tedious and painstaking

State of the Art

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- NLO ✗ no $2 \rightarrow 4$ LHC cross sections known
- ✗ need to extend range of available calculations to e.g. $pp \rightarrow W + \text{multijets}$, $t\bar{t}b\bar{b}$, $t\bar{t} + 2 \text{ jets}$, $VV + 2 \text{ jets}$ that are backgrounds to New Physics
- ✓ 4 gluons@one-loop, Ellis, Sexton, 1986, σ_{2j} , 1992
- ✓ 5 gluons@one-loop, Bern, Dixon, Kosower, 1993, σ_{3j} , 2000
- ✓ 6 gluons@one-loop, many authors, 2006 σ_{4j} , 20??
- ✗ need a more efficient way of evaluating loop contributions and constructing σ
- ✓ 4 gluons@two-loop, BDDK, AGOTY, 2001, σ_{2j} , 20??

How to calculate scattering amplitudes

1. Off-shell methods

Traditional Feynman diagram approach - big strides at recent “High Precision for Hard Processes” workshop

- ✓ improved analytic methods
- ✓ semi-numerical methods
- ✓ purely numerical methods

2. On-shell methods

Based on S-matrix ideas of 1960's but recently inspired by **Witten's** proposal to relate **perturbative gauge theory amplitudes** to **topological string theory in twistor space**

Witten, hep-th/0312171

⇒ new ways to calculate amplitudes in massless gauge theories:

Off-shell methods

Traditional Feynman diagram approach for off-shell Greens functions

- ✓ Direct link to Lagrangian
- ✓ Easy to adapt to any model
- ✓ Easy to include massive particles with/without spin
- ✓ Easy to automate
 - ⇒ tree-level packages Madgraph/Grace/CompHep/...
- ✓ Off-shell Berends-Giele recursion relations
 - ⇒ tree-level packages Alpgen/HELAC/PHEGAS/...
- ✗ Many Feynman diagrams
- ✗ Large cancellations between diagrams
- ✗ Loop amplitudes manpower intensive

Example

Multi-jet production at the LHC using HELAC/PHEGAS

Draggiotis, Kleiss, Papadopoloulos

# of jets	2	3	4	5	6	7	8
# of dist.processes	10	14	28	36	64	78	130
total # of processes	126	206	621	861	1862	2326	4342
$\sigma(nb)$	-	91.41	6.54	0.458	0.030	0.0022	0.00021
% Gluonic	-	45.7	39.2	35.7	35.1	33.8	26.6

- The number of Feynman diagrams for an n gluon process increases very quickly with n
- ⇒ for the 10 gluon amplitude there are 10,525,900 diagrams
- ⇒ Feynman diagrams very inefficient for many legs
- Control the quantum numbers of the scattering particles

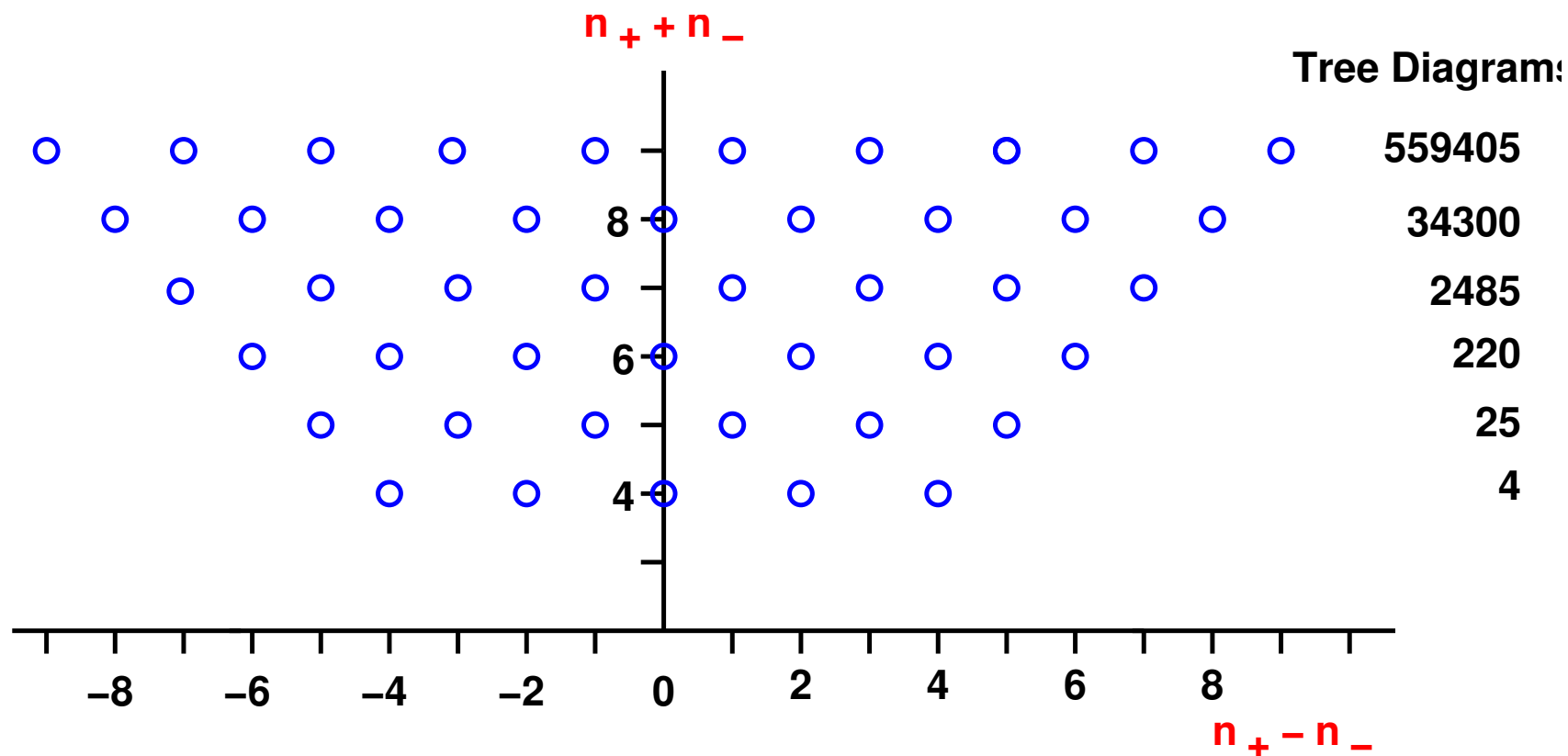
On-shell methods

- ✓ New (and puzzling) insights into field theory amplitudes
⇒ new ways to calculate amplitudes in massless gauge theories:
 - ✓ MHV rules Cachazo, Svrcek and Witten
⇒ NEW analytic results for some QCD tree amplitudes with any number of legs
 - ✓ BCF on-shell recursion relations Britto, Cachazo and Feng (and Witten)
⇒ NEW compact results for some multileg QCD tree amplitudes
 - ✓ Unitarity and cut-constructibility Bern, Dixon, Dunbar, Kosower; Britto, Cachazo and Feng; . . .
⇒ NEW analytic one-loop amplitudes in massless supersymmetric theories
 - ✓ Recursive derivation of rational terms Bern, Dixon, Kosower + Berger, Forde; Xiao, Yang, Zhu
⇒ NEW analytic one-loop amplitudes for multigluon amplitudes

Gluonic helicity amplitudes

- A gluon is a spin-1 (vector) boson like the photon and has two polarisations which we label as either positive or negative helicity (right-handed or left-handed)
- A multigluon amplitude can be characterised by the helicity of the gluons
- There will n_+ positive helicities and n_- negative helicities.
- In general, the order of helicities matters:
 $- - + + + + + \dots$ is not the same as $- + - + + + + \dots$ etc.

Gluonic helicity amplitudes



Each row describes scattering with n_+ positive helicities and n_- negative helicities.

Each circle represents an allowed helicity configuration - from all positive on the right to all negative on the left

Gluonic helicity amplitudes

For example, the result of computing the 25 diagrams for the colour-ordered five-gluon process yields

$$A_5(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

In fact, for n point colour-ordered amplitudes,

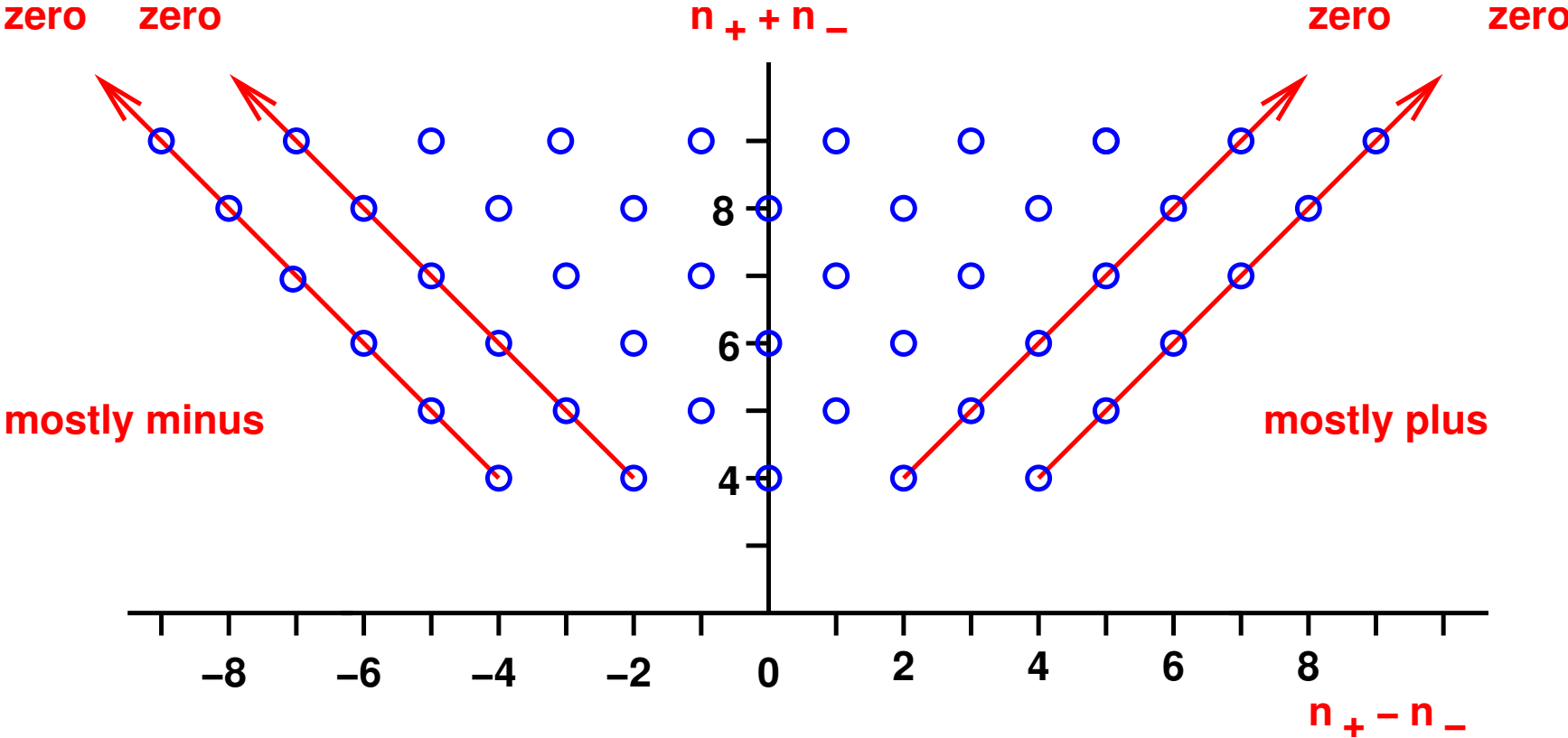
$$A_n(1^\pm, 2^+, 3^+, \dots, n^+) = 0$$

$$A_n(1^-, 2^-, 3^+, \dots, n^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

$$A_n(1^-, 2^+, 3^-, \dots, n^+) = \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

Maximally helicity violating (MHV) amplitudes

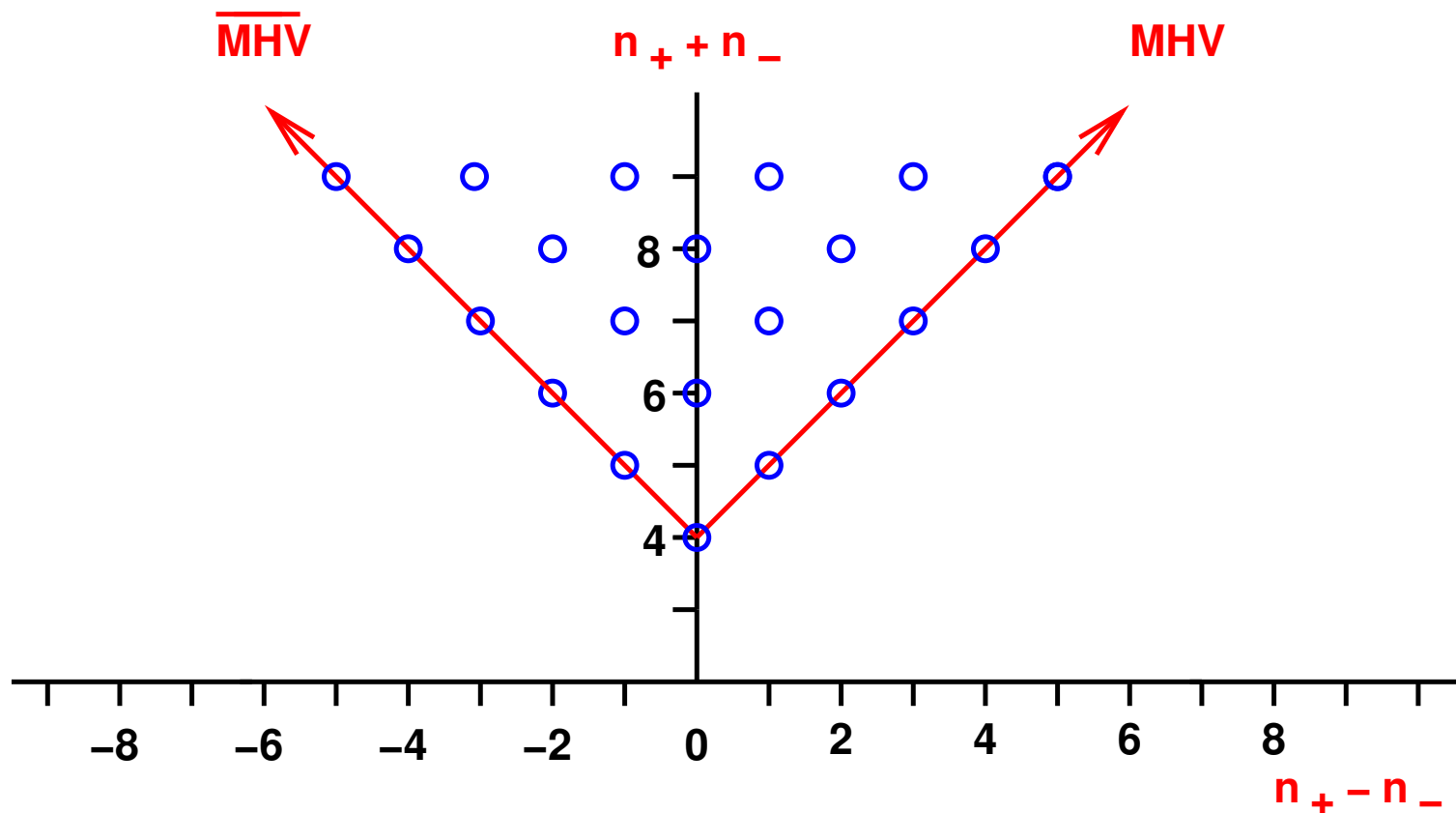
Gluonic helicity amplitudes



$$A_n(1^\pm, 2^+, 3^+, \dots, n^+) = 0$$

effective tree-level supersymmetry

Gluonic helicity amplitudes



$$A_n(1^-, 2^-, 3^+, \dots, n^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

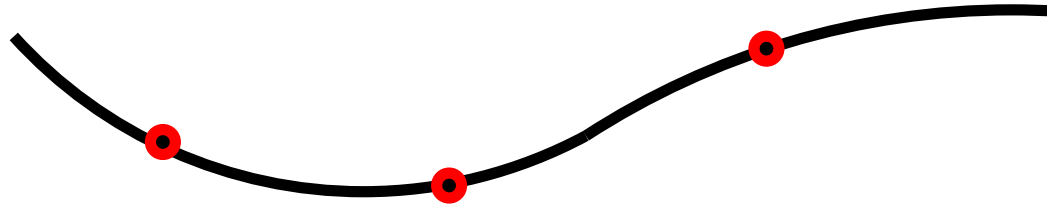
Twistor Space

Witten, hep-th/0312171

Witten observed that in twistor space external points lie on certain algebraic curves

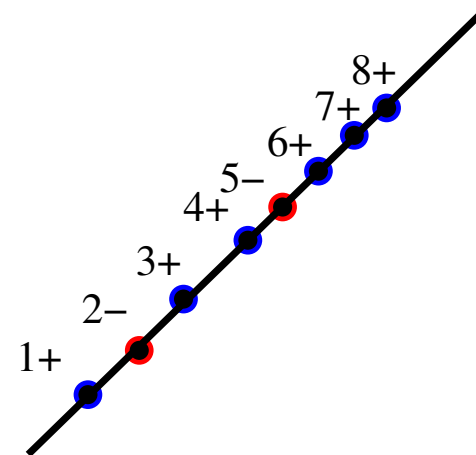
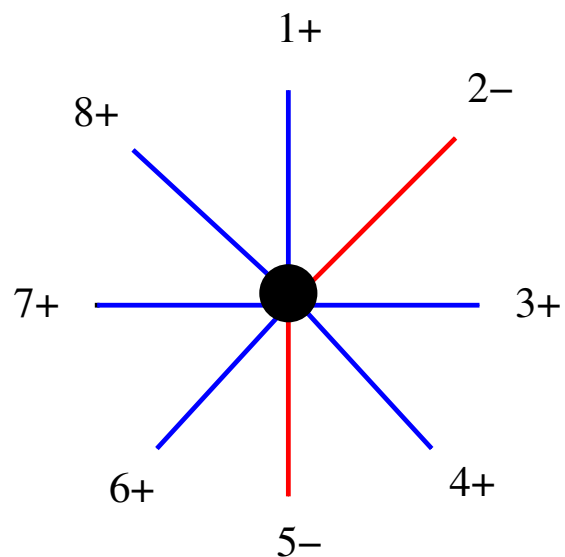
⇒ degree of curve is related to the number of negative helicities and loops

$$d = n_- - 1 + l$$

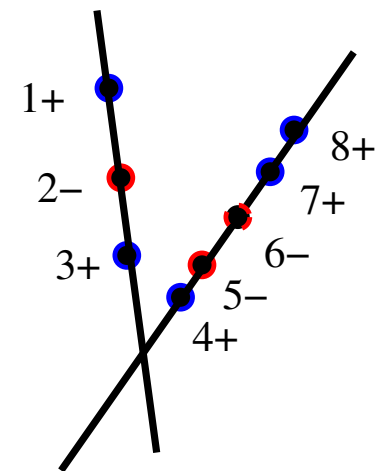
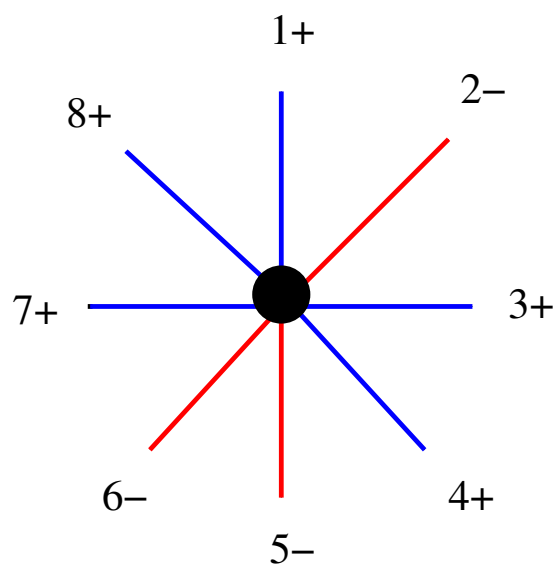


Twistor Space

MHV



NMHV



MHV rules

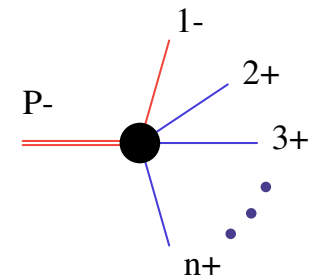
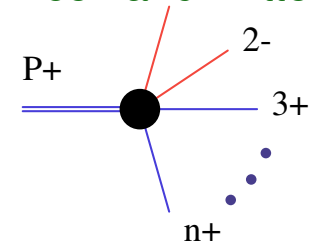
Start from **on-shell** MHV amplitude and define **off-shell** vertices

Cachazo, Svrcek and Witten

$$V(1^-, 2^-, 3^+, \dots, n^+, P^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \cdots \langle n-1n \rangle \langle nP \rangle \langle P1 \rangle}$$

and

$$V(1^-, 2^+, 3^+, \dots, n^+, P^-) = \frac{\langle 1P \rangle^4}{\langle 12 \rangle \cdots \langle n-1n \rangle \langle nP \rangle \langle P1 \rangle}$$



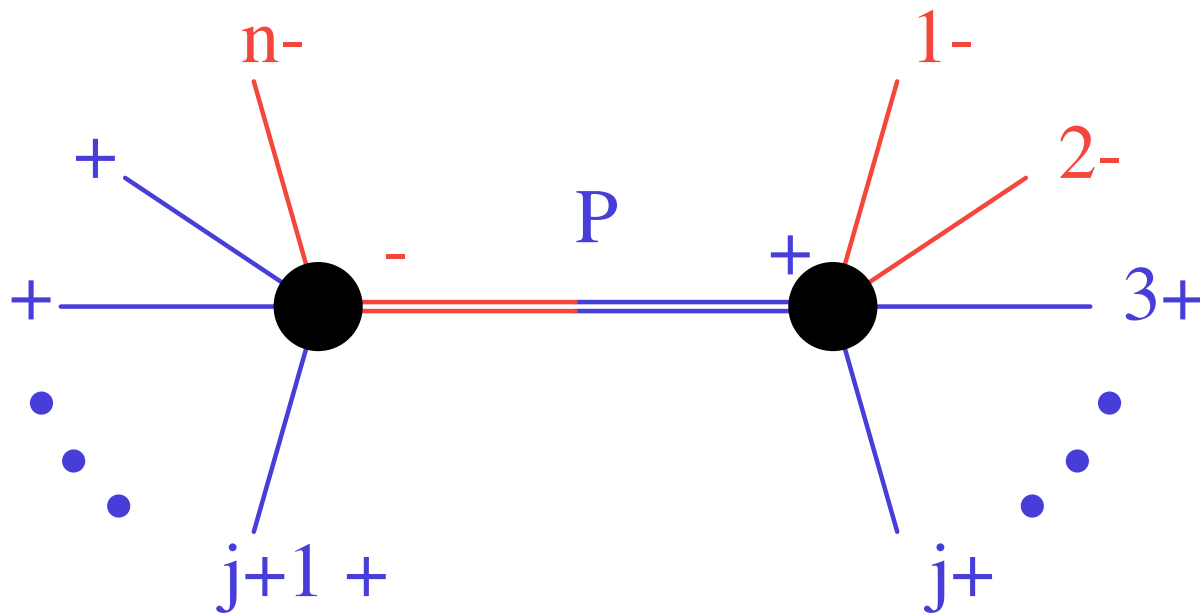
Crucial step is **off-shell** continuation $P^2 \neq 0$:

$$\langle iP \rangle = \frac{\langle i^- | P | \eta^- \rangle}{[P\eta]} = \sum_j \frac{\langle i^- | j | \eta^- \rangle}{[P\eta]}$$

where $P = \sum_j j$ and η is lightlike auxiliary vector

MHV rules

Must connect up a positive helicity off-shell line to a negative helicity off-shell line with a scalar propagator



Connecting two MHV's \Rightarrow amplitude with 3 negative helicities
Connecting three MHV's \Rightarrow amplitude with 4 negative helicities
etc.

Example: six gluon scattering

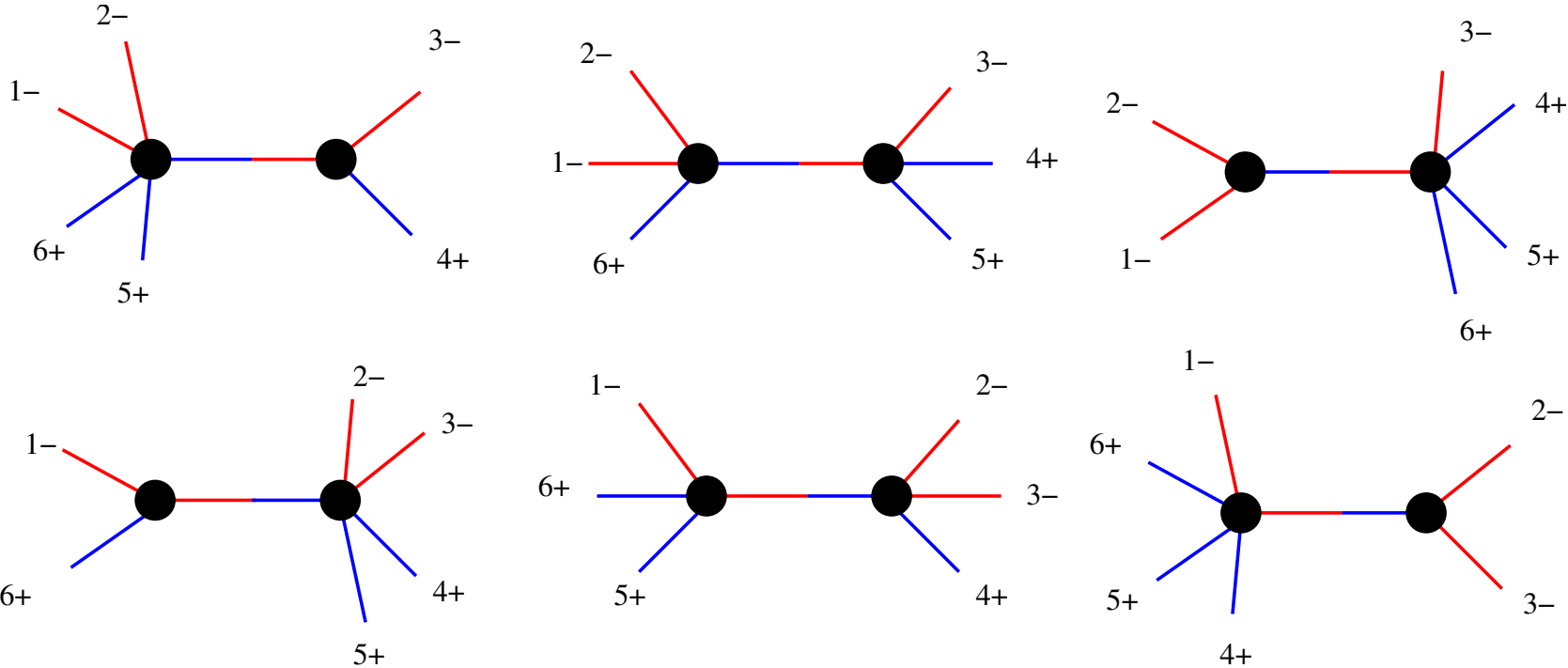
As an example, let's use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

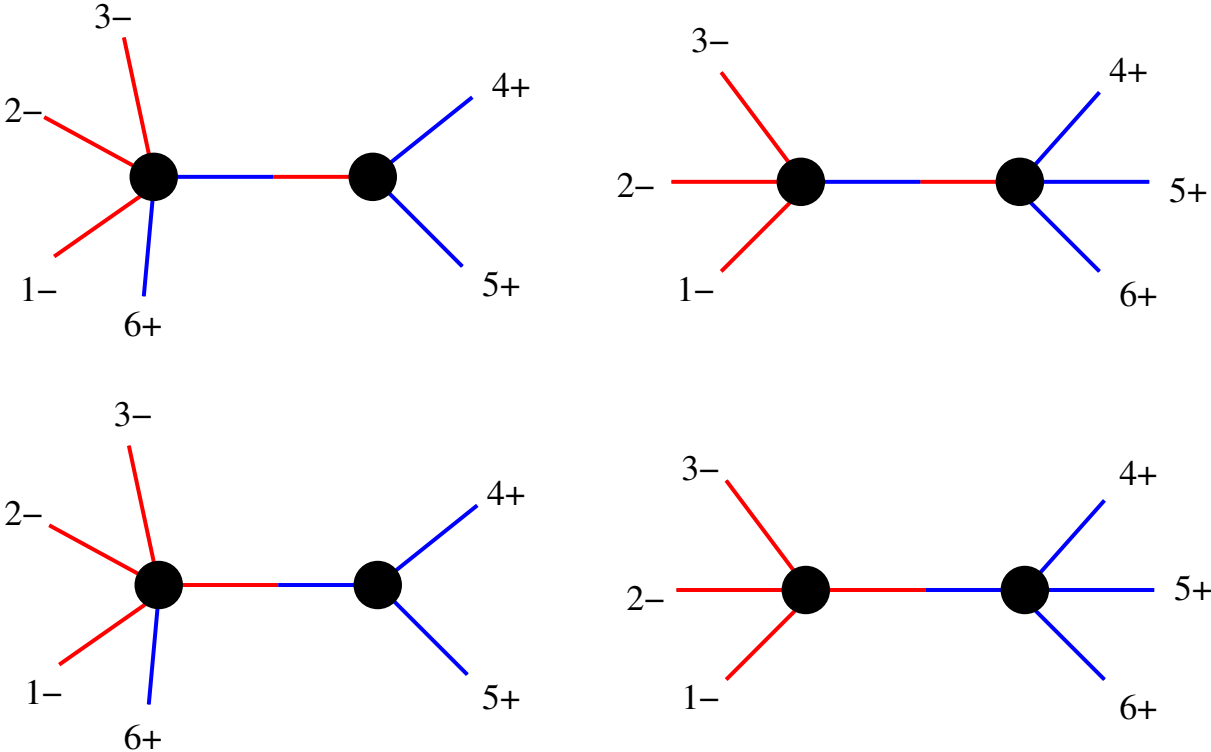
Example: six gluon scattering

There are six MHV graphs



Example: six gluon scattering

Some graphs are not allowed e.g.



Example: six gluon scattering

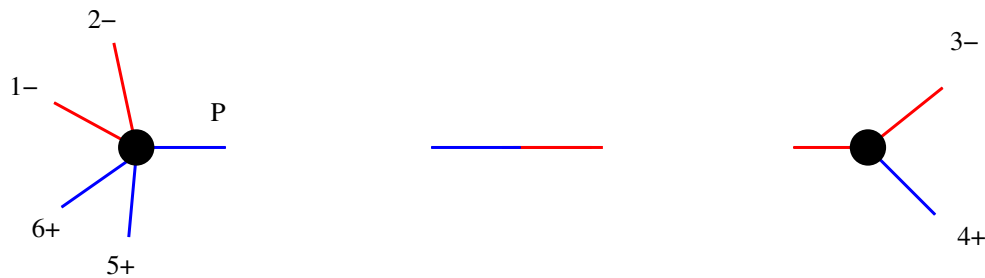
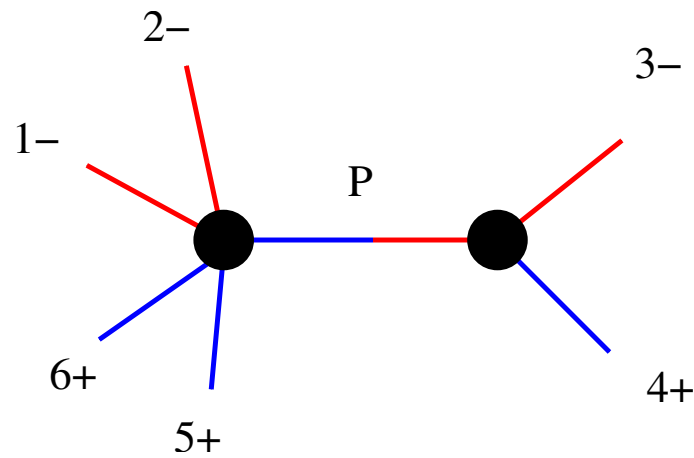
As an example, let's use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

Step 2 Apply MHV rules to each diagram

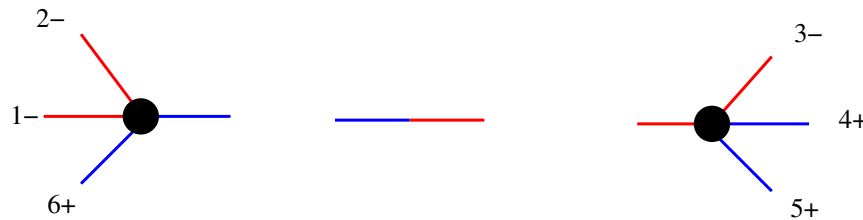
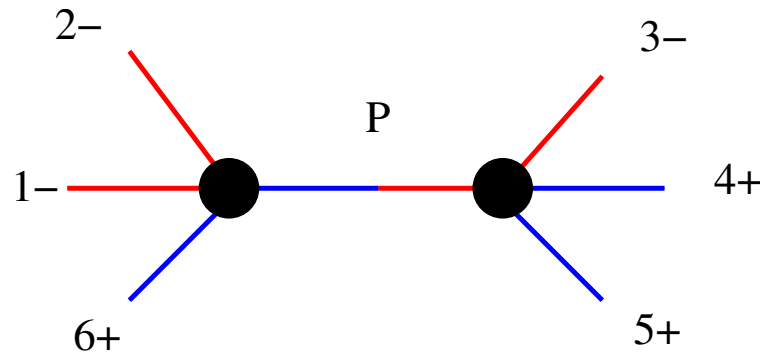
Example: six gluon scattering: diagram 1



$$\frac{\langle 12 \rangle^4}{\langle 56 \rangle \langle 61 \rangle \langle 12 \rangle \langle 2|P|\eta \rangle \langle 5|P|\eta \rangle} \times \frac{1}{s_{34}} \times \frac{\langle 3|P|\eta \rangle^4}{\langle 34 \rangle \langle 4|P|\eta \rangle \langle 3|P|\eta \rangle}$$

with $P = 3 + 4 = -(1 + 2 + 5 + 6)$

Example: six gluon scattering: diagram 2



$$\frac{\langle 12 \rangle^4}{\langle 61 \rangle \langle 12 \rangle \langle 2|P|\eta \rangle \langle 6|P|\eta \rangle} \times \frac{1}{s_{345}} \times \frac{\langle 3|P|\eta \rangle^4}{\langle 34 \rangle \langle 45 \rangle \langle 5|P|\eta \rangle \langle 3|P|\eta \rangle}$$

with $P = 3 + 4 + 5 = -(1 + 2 + 6)$

Example: six gluon scattering

As an example, let's use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

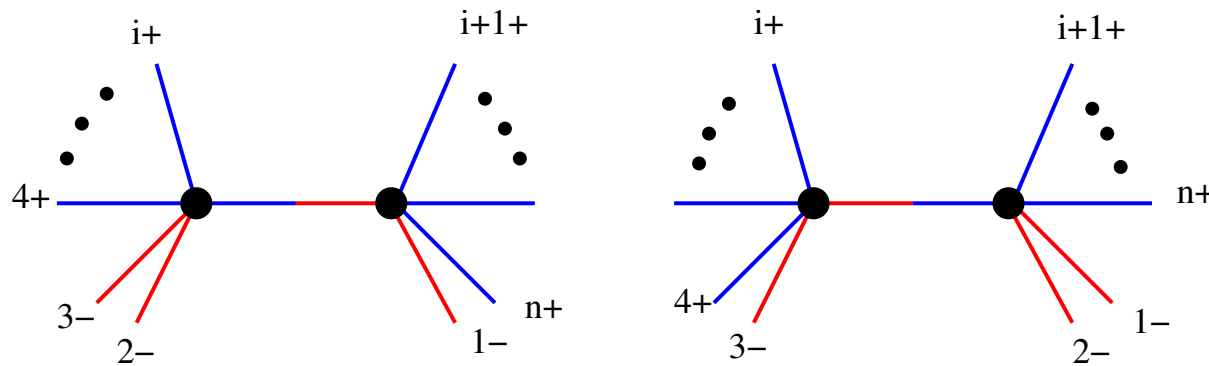
- Step 1 Draw all the allowed MHV diagrams
- Step 2 Apply MHV rules to each diagram
- Step 3 Add up diagrams and check η independence

Next-to MHV amplitude for n gluons

Simplest case: $A_n(1^-, 2^-, 3^-, 4^+, \dots, n^+)$

$2(n - 3)$ graphs

Cachazo, Svrcek and Witten



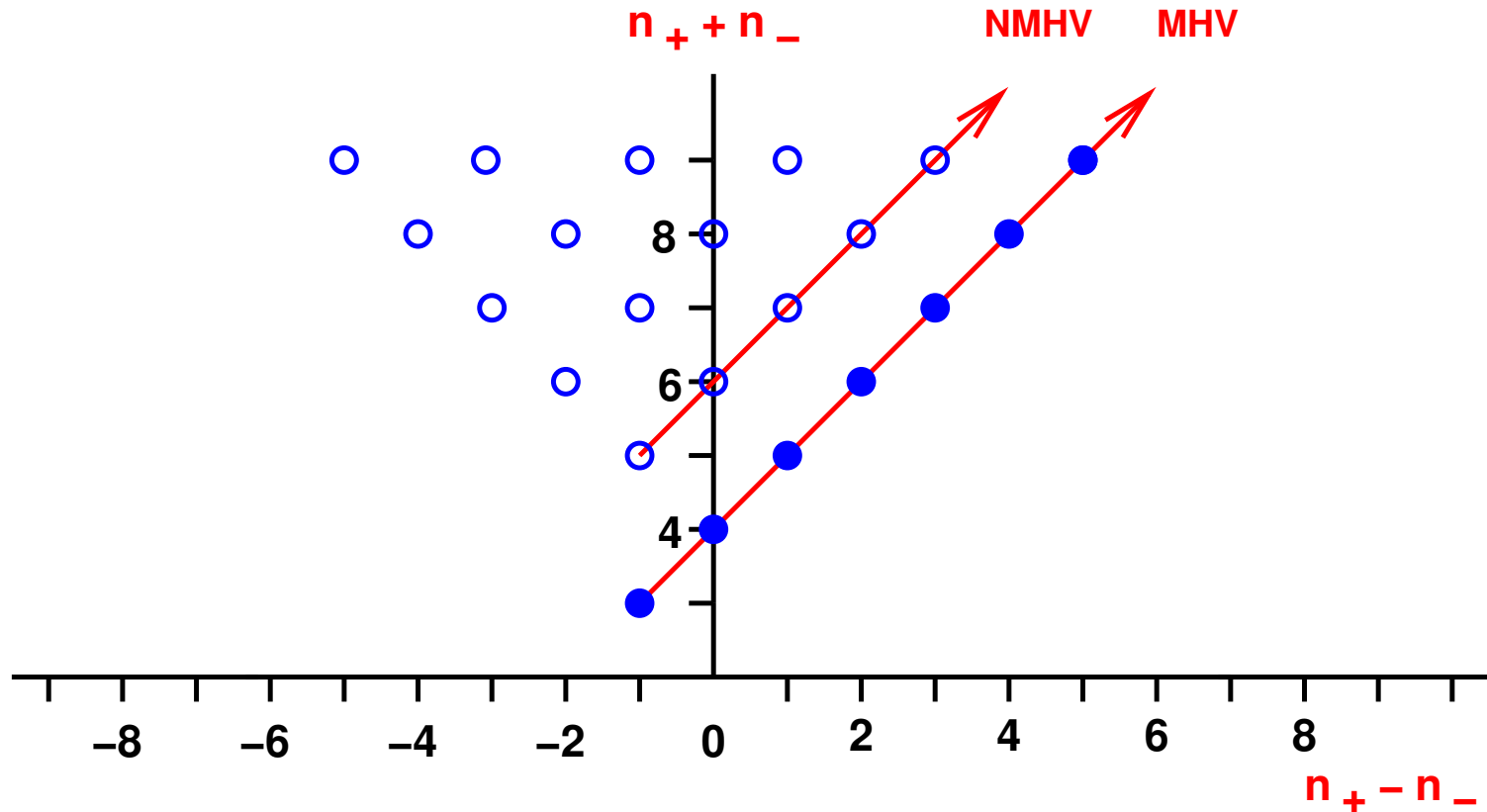
$$\begin{aligned}
 A &= \sum_{i=3}^{n-1} \frac{\langle 1|(2, i)|\eta\rangle^3}{\langle (i+1)|(2, i)|\eta\rangle \langle i+1i+2\rangle \dots \langle n1\rangle} \frac{1}{s_{2,i}^2} \frac{\langle 23\rangle^3}{\langle 2|(2, i)|\eta\rangle \langle 34\rangle \dots \langle i|(2, i)|\eta\rangle} \\
 &+ \sum_{i=4}^n \frac{\langle 12\rangle^3}{\langle 2|(3, i)|\eta\rangle \langle (i+1)|(3, i)|\eta\rangle \dots \langle n1\rangle} \frac{1}{s_{3,i}^2} \frac{\langle 3|(3, i)|\eta\rangle^3}{\langle 34\rangle \dots \langle i-1i\rangle \langle i|(3, i)|\eta\rangle}.
 \end{aligned}$$

where $(k, i) = k + \dots + i$ is the off-shell momentum

\Rightarrow Lorentz invariant and gauge invariant expressions

Generating all the tree amplitudes

Amplitudes with $i-$ and $j+$ helicities



- MHV rules always adds one negative helicity and any number of positive helicities
 \Rightarrow maps out all allowed tree amplitudes

Other processes

MHV rules have been generalised to many other processes

✓ with massless fermions - quarks, gluinos

Georgiou and Khoze; Wu and Zhu; Georgiou, EWNG and Khoze

✓ with massless scalars - squarks

Georgiou, EWNG and Khoze; Khoze

✓ with an external Higgs boson

Dixon, EWNG, Khoze; Badger, EWNG, Khoze

✓ with an external weak boson

Bern, Forde, Kosower and Mastrolia

Has provided **new** analytic results for n -particle amplitudes

Also useful for studying infrared properties of amplitudes

Birthwright, EWNG, Khoze and Marquard

Links with lagrangian

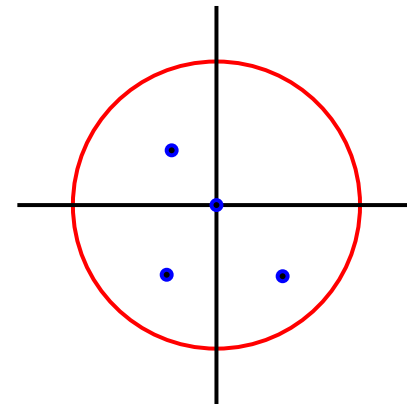
Mansfield; Etle, Morris

BCFW on-shell recursion relations

Britto, Cachazo, Feng, Witten; Roiban, Spradlin, Volovich

Based on elementary complex analysis - **Cauchy Integral Formula**

$$\frac{1}{2\pi i} \oint \frac{dz}{z} A(z) = \text{sum of residues}$$



provided that $A(z) \rightarrow 0$ as $z \rightarrow \infty$

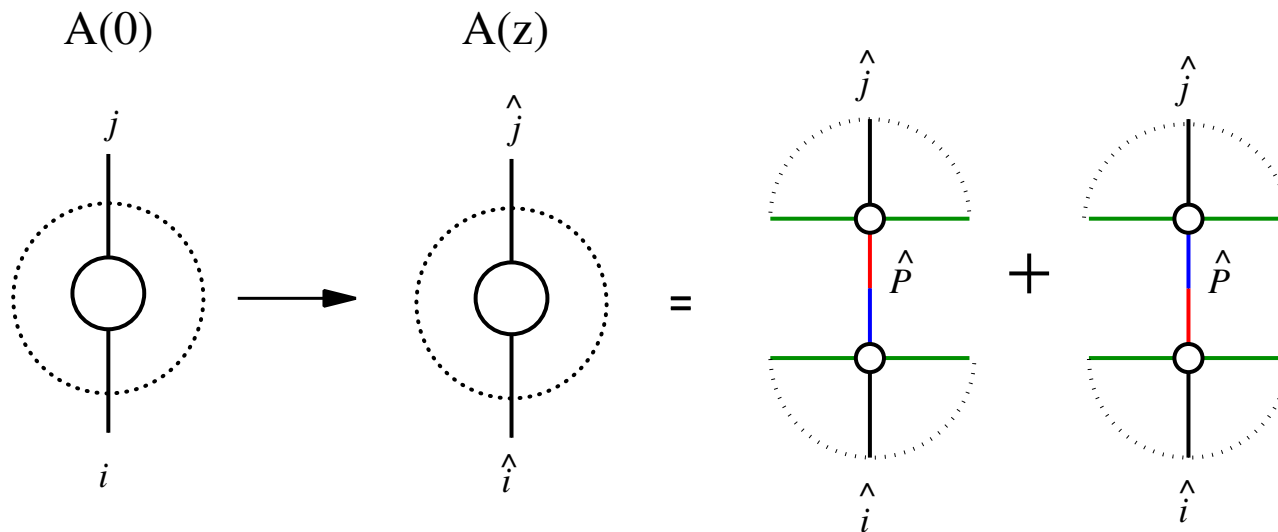
$$\text{sum of residues} = A(0) + \dots$$

Simple enough, but how is this related to scattering amplitudes?

BCFW on-shell recursion relations

Britto, Cachazo, Feng; Roiban, Spradlin, Volovich

Lets consider an n particle amplitude $A(0)$.



hatted momenta are shifted to put on-shell

$$\hat{i} = i + z\eta, \quad \hat{j} = j - z\eta, \quad \hat{P} = P + z\eta$$

\Rightarrow each vertex is an **on-shell** amplitude

BCFW recursion relations

- It turns out that the shift η is not a momentum, but

$$\eta = \lambda_i \tilde{\lambda}_j \quad OR \quad \eta = \lambda_j \tilde{\lambda}_i$$

- The parameter z is fixed by $\hat{P}^2 = 0$

$$z = \frac{P^2}{\langle j|P|i \rangle}$$

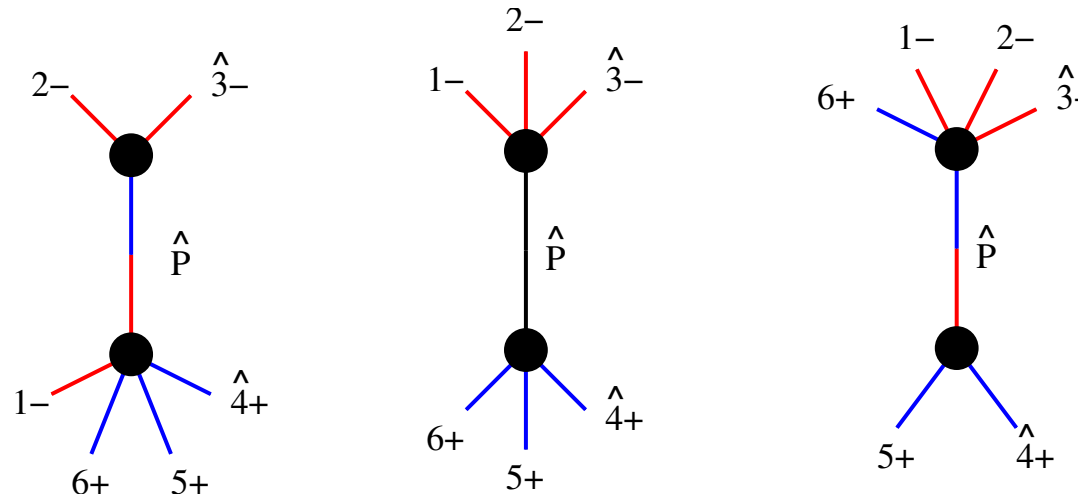
- Easy to prove that by complex analysis based on fact that only simple poles in z occur and that $A(z)$ vanishes as $z \rightarrow \infty$

Britto, Cachazo, Feng and Witten

- Requires on-shell three-point vertex contributions - both MHV and $\overline{\text{MHV}}$

BCFW - six gluon example

If we select 3 and 4 to be the special gluons, there are only three diagrams (for any helicities)



For this helicity assignment, the middle diagram is zero!

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

$$= \frac{1}{\langle 5|\cancel{3} + \cancel{4}|2\rangle} \left(\frac{\langle 1|\cancel{2} + \cancel{3}|4\rangle^3}{[23][34]\langle 56\rangle\langle 61\rangle s_{234}} + \frac{\langle 3|\cancel{4} + \cancel{5}|6\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle s_{345}} \right)$$

Extremely compact analytic results for up to 8 gluons

Other processes

BCF recursion relations have been generalised to other processes

- ✓ with massless fermions - quarks, gluinos

Luo and Wen

- ✓ gravitons

Bedford, Brandhuber, Spence and Travaglini; Cachazo and Svrcek

There is nothing (in principle) to stop this approach being applied to particles with mass.

- ✓ massive coloured scalars

Badger, EWNG, Khoze and Svrcek

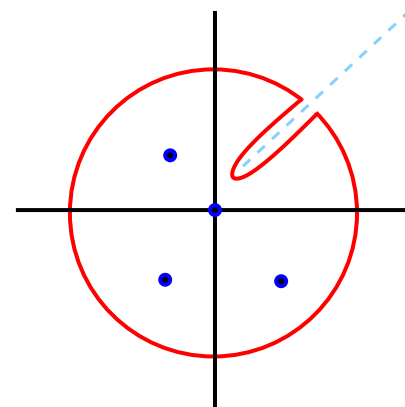
- ✓ massive vector bosons and heavy quarks

Badger, EWNG and Khoze

One loop amplitudes

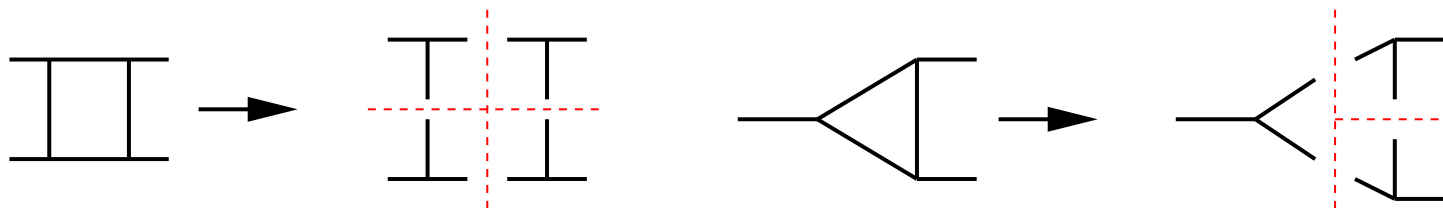
- Loop amplitudes contain **both** poles and cuts

$$A_n^1 \sim (\text{poly})\text{logs} + \text{rational}$$



e.g. $\log(x)$ has cut for negative x

- Cut** contributions are fully constructible by using **unitarity**
 - Cut lines are **on-shell**



Unitarity Bootstrap

- 1 **Cut** contributions are fully constructible by using **4-dimensional unitarity** and exploiting the tree-level helicity amplitudes known in 4-dimensions

Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng

- 2 **Pole** contributions can be constructed using BCFW type recursion and knowledge of factorisation properties

Berger, Bern, Dixon, Forde, Kosower, ...

or by direct evaluation of (ultraviolet) parts of Feynman graphs

Xiao, Yang, Zhu; Binoth, Guillet, Heinrich; Ossola, Papadopoulos, Pittau, ...

- 3 **Both** contributions can be reconstructed simultaneously using ***d*-dimensional unitarity**

Anastasiou, Britto, Feng, Kunzst, Mastrolia

$$1 + 2 \equiv 3$$

SUSY QCD loops

- So far, supersymmetry was not a major factor - tree level amplitudes same for $\mathcal{N} = 4$ and $\mathcal{N} = 1$ and QCD
- Not true at the loop level due to circulating states

$$\begin{aligned}
 A_n^{\mathcal{N}=4} &= A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]} \\
 A_n^{\mathcal{N}=1, \text{chiral}} &= A_n^{[1/2]} + A_n^{[0]} \\
 A_n^{\text{glue}} &= A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1, \text{chiral}} + A_n^{[0]}
 \end{aligned}$$

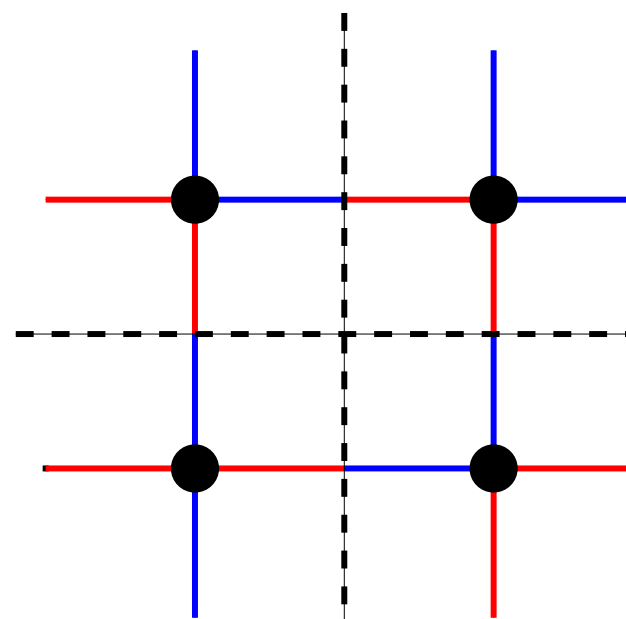
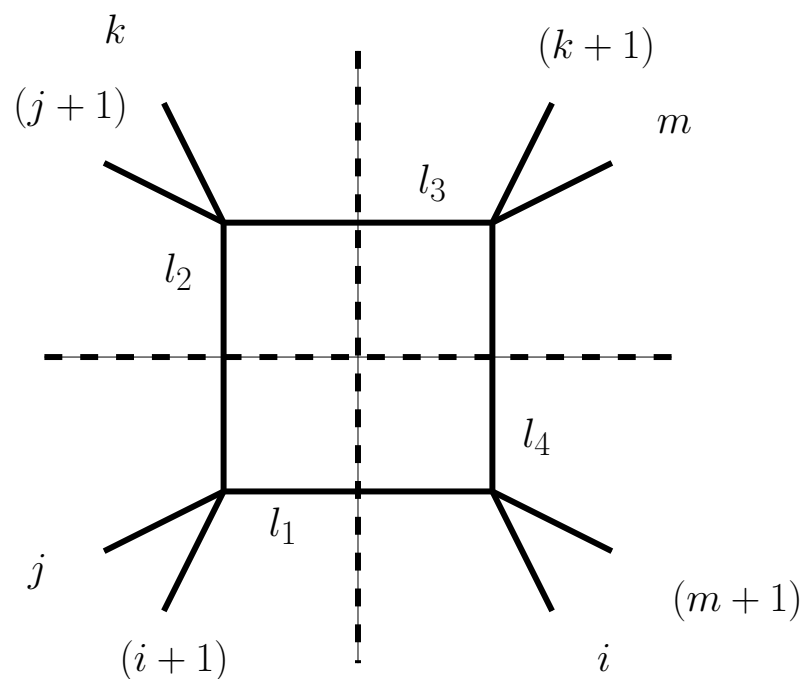
- $\mathcal{N} = 4$ and $\mathcal{N} = 1$ one-loop amplitudes are fully constructible from their 4-dimensional cuts
- For $\mathcal{N} = 4$ all amplitudes are a linear combination of known box integrals

$$\mathbf{A_n} = \Sigma + \mathbf{a} \text{ [diagram]} + \mathbf{b} \text{ [diagram]} + \mathbf{c} \text{ [diagram]} + \mathbf{d} \text{ [diagram]} + \mathbf{e} \text{ [diagram]} + \mathbf{f} \text{ [diagram]}$$

Generalised unitarity

- Coefficients of box graphs fixed uniquely - one at a time - by taking the four particle cut. No integrals left to do!

Britto, Cachazo, Feng

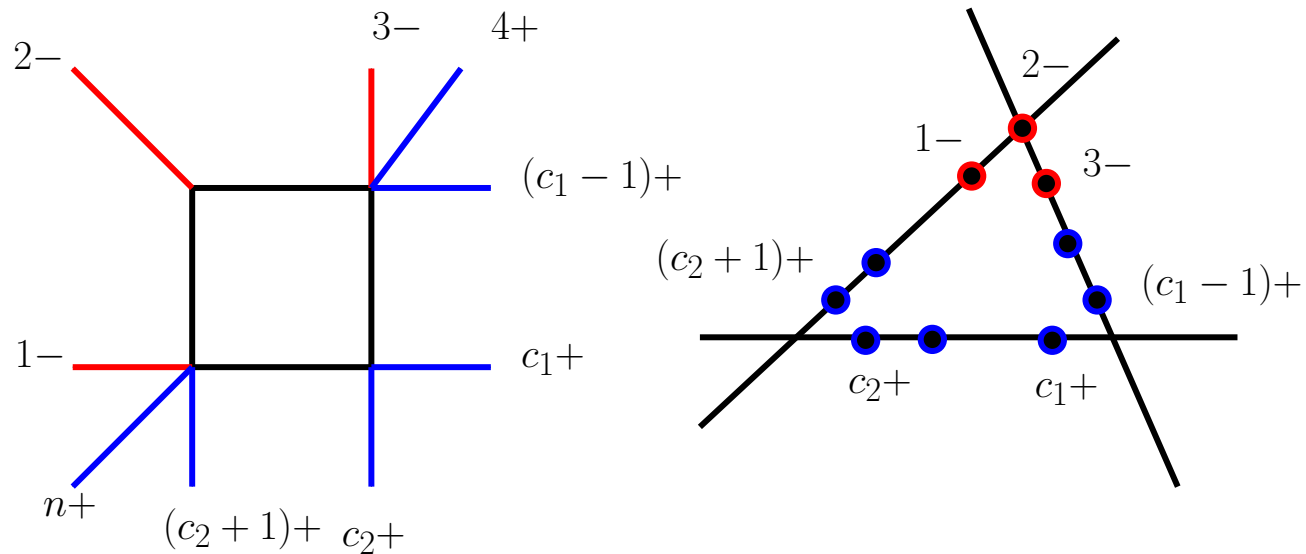


- Coefficients of boxes are products of MHV's subject to constraint that propagators are on-shell

Twistor space interpretation

- Coefficients of boxes have very interesting structures.

Bern, Del Duca, Dixon, Kosower; Britto, Cachazo, Feng

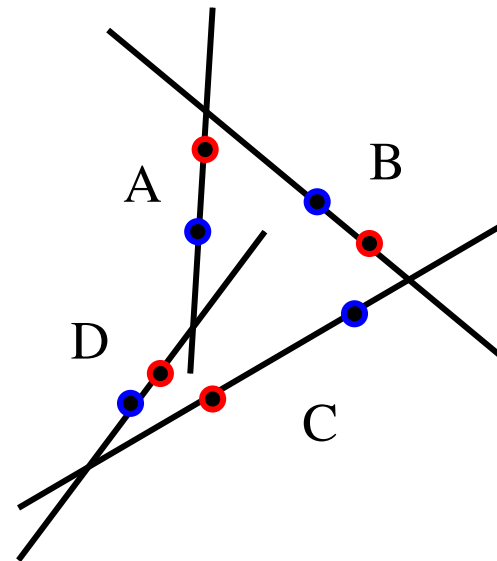
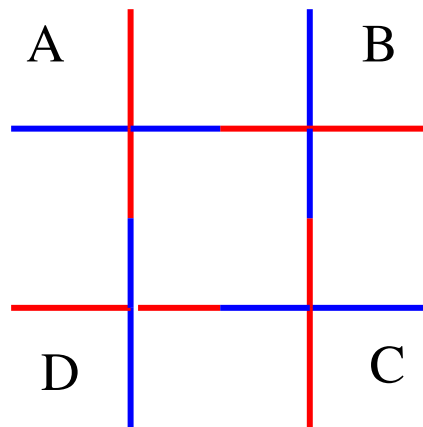


Twistor space interpretation

- Four mass box first appears in eight-point amplitude with four negative and four positive helicities

Bern, Dixon, Kosower

e.g.



- Why this is so is still not fully understood

QCD loops

✗ QCD amplitudes more complicated because they are **not** 4-dimensional cut constructible.

Rational contribution not probed by 4-d cut

✗ All plus and almost all plus amplitudes no longer zero - but pure rational functions. Not protected by SWI.

✓ Rational parts of infrared divergent amplitudes computed using

✓ on-shell recursion relation

Bern, Dixon and Kosower

Recursion relations complicated by double pole terms and boundary terms

✓ Direct Feynman diagram evaluation of rational part

Xiao, Yang, Zhu

✓ d -dimensional cuts

Anastasiou, Britto, Feng, Kunszt, Mastrolia

Six gluon amplitude

✓ Analytic computation

Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

Amplitude	$\mathcal{N} = 4$	$\mathcal{N} = 1$	$\mathcal{N} = 0$ (cut)	$\mathcal{N} = 0$ (rat)
- - + + + +	BDDK (94)	BDDK (94)	BDDK (94)	BDK (94)
- + - + + +	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
- + + - + +	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
- - - + + +	BDDK (94)	BDDK (94)	BBDI (05), BFM (06)	BBDFK (06), XYZ (06)
- - + - + +	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)
- + - + - +	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)

✓ Numerical evaluation Ellis, Giele, Zanderighi (06)

Summary - I

- ✓ **On-shell** techniques are a very exciting and rapidly developing field
- ✓ **MHV** rules for tree-level
Very simple way of deriving n -point amplitudes for massless partons
- ✓ **BCFW** recursion relations for tree-level
Very powerful method for deriving amplitudes for both massless and massive particles
- ✗ Berends-Giele recursion still looks to be numerically faster
- ✓ **Generalised unitarity** and one-loop amplitudes
SUSY amplitudes cut constructible - coefficients of loop integrals can be *read off* from graphs
QCD amplitudes contain cut-non constructible parts. These simple pole terms can be attacked using the BCFW relations

Bern, Dixon, Kosower

Or by direct evaluation using Feynman diagrams

Xiao, Yang, Zhu

Summary - II

- ✓ New methods already competitive with traditional methods for loop amplitudes with massless particles - gluons, quarks
- ✓ Will definitely see all six parton one-loop amplitudes in next few months
- ✗ Not necessarily the most interesting phenomenologically
- ? Will new methods be useful for amplitudes with heavy particles - top quarks, susy particles, Higgs bosons, vector bosons
- ✓ In principle heavy particles not a problem - but certainly a complication.
- ✓ **yes** for one vector boson plus multiparton e.g. **$V + \text{multijet}$**
- ✓ **probable** for two vector boson plus multiparton e.g. **$VV + \text{multijet}$**
- ? much more difficult for **$pp \rightarrow t\bar{t}b\bar{b}$**
 - ✓ ✓ Expect more progress in 2007

SPARE SLIDES

Collider Physics

1. Predictions for multiparticle final states that occur at high rate and form background to **New Physics**

High multiplicity, but low order - typically **LO** or **NLO**

For example, $pp \rightarrow V + 4 \text{ jets}$ is background to $pp \rightarrow t\bar{t}$ and other new physics.

2. Precise predictions for hard pp processes involving “standard particles” like W , Z , jets, top, Higgs, ..

Low multiplicity, but high order - **NNLO** is emerging standard

For example, Drell Yan cross section.

State of the Art

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- NNLO ✓ (inclusive) Drell-Yan and Higgs total cross sections – Anastasiou, Dixon, Melnikov, Petriello
- ✓ (inclusive) Drell-Yan and Higgs rapidity distributions – Anastasiou, Dixon, Melnikov, Petriello
- ✓ NNLO evolution – Moch, Vogt, Vermaseren
- ✗ need full set of NNLO observables for global fit. DIS and Drell-Yan will not be enough

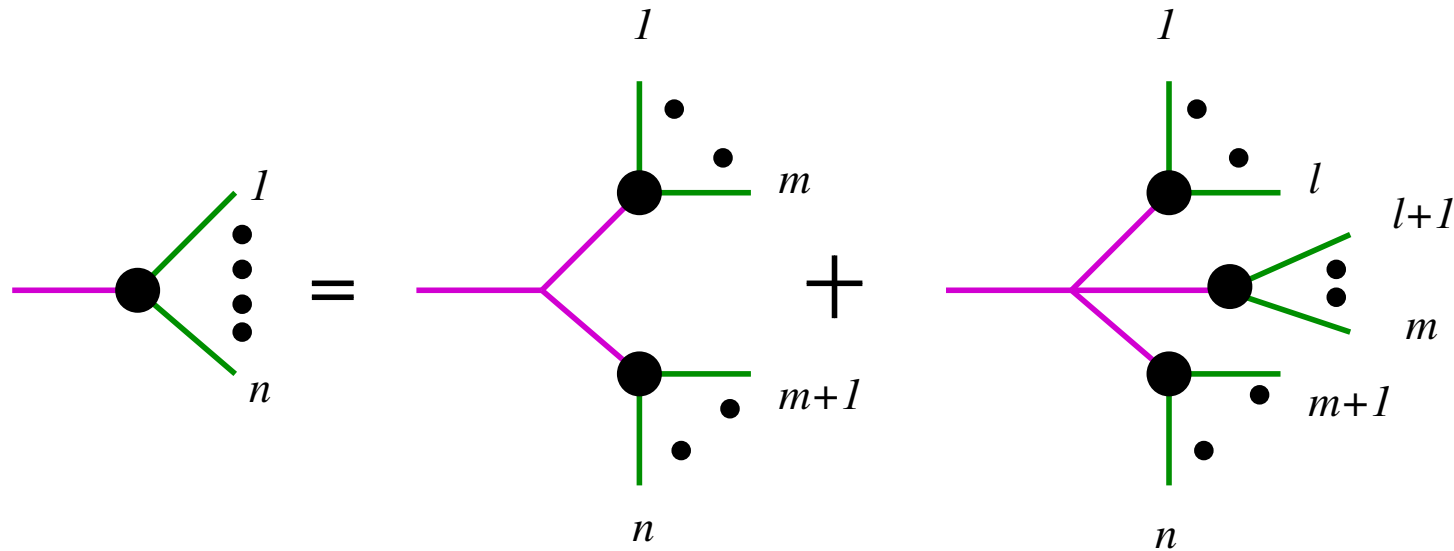
State of the Art

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- NNLO** ✓ want to calculate $2 \rightarrow 2$ to few percent accuracy and use as standard candle to determine **pdfs** and α_s more accurately
- ✓ with global pdf fit, gives impact on **all** observables
- ✗ still not available

Berends-Giele : Off-shell recursion relations

Full amplitudes can be built up from simpler amplitudes with fewer particles



Purple gluons are off-shell, green gluons are on-shell.
This is a recursion relation built from off-shell currents.

Berends, Giele

Particularly suited to numerical solution

ALPGEN, HELAC/PHEGAS

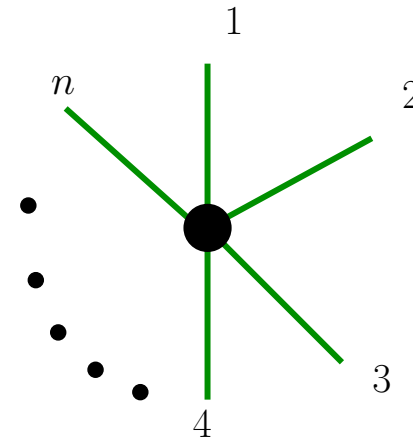
Common methods : Colour Ordered Amplitudes

$$\mathcal{A}_n(1, \dots, n) = \sum_{perms} Tr(T^{a_1} \dots T^{a_n}) A_n(1, \dots, n)$$

Colour-stripped amplitudes A_n : cyclically ordered

Order of external gluons fixed

The subamplitudes A_n have nice properties in the infrared limits.



Can reconstruct the full amplitude \mathcal{A}_n from A_n .
In the large N limit,

$$|\mathcal{A}_n(1, \dots, n)|^2 \sim N^{n-2} \sum_{perms} |A_n(1, \dots, n)|^2$$

Twistor Space

Penrose, 1967

Amplitudes in **twistor space** obtained by Fourier transform with respect to positive helicity spinors,

$$\tilde{\lambda}_{\dot{a}} = i \frac{\partial}{\partial \mu^{\dot{a}}}, \quad \mu^{\dot{a}} = i \frac{\partial}{\partial \tilde{\lambda}_{\dot{a}}}$$

Momentum conservation yields

$$\delta \left(\sum k_j \right) = \int d^4 x \exp \left(i \sum_j x \cdot k_j \right) = \int d^4 x \exp \left(i x^{a\dot{a}} \sum_j \lambda_{ja} \tilde{\lambda}_{j\dot{a}} \right)$$

so that the amplitude in twistor space is

$$\tilde{A}(\lambda_i, \mu_i) = \int d^4 x \int \prod_i \frac{d^2 \tilde{\lambda}_i}{(2\pi)^2} \exp \left(i \sum_j (\mu_j^{\dot{a}} + x^{a\dot{a}} \lambda_{ja}) \tilde{\lambda}_{j\dot{a}} \right) A(\lambda_i, \tilde{\lambda}_i)$$

Spinor Helicity Formalism

- In Weyl (chiral) representation, each helicity state is represented by a bi-spinor ($a = 1, 2$)

$$\begin{aligned}u_+(p) &= \lambda_{pa}, & u_-(p) &= \tilde{\lambda}_p^{\dot{a}}, \\ \overline{u_+(p)} &= \tilde{\lambda}_{p\dot{a}}, & \overline{u_-(p)} &= \lambda_p^a\end{aligned}$$

so that

$$\begin{aligned}\langle ij \rangle &= \overline{u_-(p_i)} u_+(p_j) = \lambda_i^a \lambda_{ja} = \epsilon_{ab} \lambda_i^a \lambda_j^b \\ [ij] &= \overline{u_+(p_i)} u_-(p_j) = \tilde{\lambda}_{i\dot{a}} \tilde{\lambda}_j^{\dot{a}} = -\epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}\end{aligned}$$

- We can write massless vector

$$p_{a\dot{a}} \equiv p_\mu \sigma_{a\dot{a}}^\mu = \lambda_{pa} \tilde{\lambda}_{p\dot{a}}$$

Spinor Helicity Formalism

- Polarisation vectors for particle i :

$$\epsilon_{ia\dot{a}}^- = \frac{\lambda_{ia}\tilde{\eta}_{\dot{a}}}{[\tilde{\lambda}_i\tilde{\eta}]}, \quad \epsilon_{ia\dot{a}}^+ = \frac{\eta_a\tilde{\lambda}_{i\dot{a}}}{\langle\eta\lambda_i\rangle}$$

- For **real** momenta in Minkowski space,

$$\tilde{\lambda} = \lambda^*$$

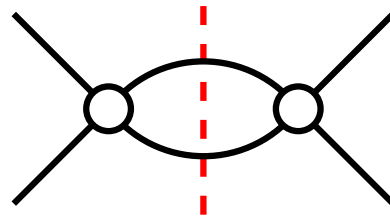
$$\langle ij \rangle^* = -[ij]$$

- For space-time signature $(+, +, -, -)$, $\tilde{\lambda}, \lambda$ are real and independent
- Amplitudes are functions of the λ_i and $\tilde{\lambda}_i$

4-d cut constructible parts

At least three different methods - all based on connecting on-shell 4-dimensional vertices

Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng; Brandhuber, Spence, Travaglini



✓ Reconstruct coefficients of basis set of integrals - boxes, triangles, bubbles

1. Classic double cut + collinear factorisation (triple cut)

Bern, Dixon, Dunbar, Kosower (94)

2. Generalised (quadruple cut) unitarity and holomorphic anomaly

Britto, Cachazo, Feng

✓ Reconstruct full amplitude by doing phase space and dispersion integrals

Brandhuber, Spence, Travaglini