LECTURE

RETURN TO OUR BASIC CONSERVATION EQN. (1)

$$\boxed{\frac{d}{dt}\langle\psi|Q|\psi\rangle = \frac{i}{\hbar}\langle\psi|[H,Q]|\psi\rangle}$$

SO FAR STUDIED CONSEQUENCES FOR SIMPLEST CASE $Q = 1$

- NOW EXPLORE MORE GENERALLY

(AS USUAL TAKE $H = \frac{p^2}{2m} + V(x)$)

i) <u>TOTAL ENERGY IS CONSERVED</u> BECAUSE

$$[H,H] = HH - HH = 0$$

SO $\frac{d}{dt}\langle\psi|H|\psi\rangle = 0$ FOR ALL $|\psi\rangle$

ii) NOW CONSIDER MOM'M p

$$[H,p] = \left[\frac{p^2}{2m} + V(x), p\right]$$

$$= \left[\frac{p^2}{2m}, p\right] + [V(x), p]$$

$$= \frac{1}{2m}\underbrace{(p^2p - pp^2)}_{0} + [V(x), p]$$

$$= [V(x), p]$$

BUT $[V(x), p] = [V(x), -i\hbar\, \partial/\partial x]$

TO WORK THIS OUT <u>PUT ARBITRARY FN ON</u>
<u>RHS</u>

$$[V(x), p]\, f(x) = \left\{ V(x)\left(-i\hbar \frac{\partial}{\partial x}\right) + i\hbar \frac{\partial}{\partial x} V(x) \right\} f(x)$$

$$= \cancel{-i\hbar V(x) \frac{\partial f}{\partial x}} + i\hbar \frac{\partial V}{\partial x} f + \cancel{i\hbar V \frac{\partial f}{\partial x}}$$

$$= i\hbar \frac{\partial V}{\partial x} . f$$

$$\Rightarrow \quad [V(x), p] = i\hbar \frac{\partial V}{\partial x} \qquad \underline{\text{NOT ZERO}}$$

THEREFORE

$$[H, p] = i\hbar \frac{\partial V}{\partial x} \qquad \text{AND}$$

$$\boxed{\frac{d\langle p \rangle}{dt} = -\left\langle \frac{\partial V}{\partial x} \right\rangle} \qquad (*)$$

NOTE SIMILARITY OF (*) TO CLASSICAL
EQN OF MOTION

(*) SHOWS THAT $d\langle p \rangle / dt$ CANNOT BE ZERO
<u>UNLESS</u> $\partial V/\partial x = 0$. BUT $\partial V/\partial x = 0$
SAYS $V =$ CONST, AND FIND LINEAR MOM'M

CONSERVATION ONLY WHEN H INDEP'T OF POSITION (SYSTEM INDEP'T OF POSITION)

iii) NOW TAKE $Q = x$

$$[H, x] = \left[\frac{p^2}{2m} + V, x\right]$$

$$= \left[\frac{p^2}{2m}, x\right] + \underbrace{[V(x), x]}_{\substack{V(x)x - xV(x) \\ = 0}}$$

USEFUL TO HAVE EXPRESSION FOR $[A^2, B]$

$$[A^2, B] = A^2B - BA^2$$

$$= AAB - \underbrace{(ABA - ABA)}_{0} - BAA$$

$$= A(AB - BA) + (AB - BA)A$$

$$= A[A, B] + [A, B]A$$

APPLYING THIS

$$[H, x] = \frac{1}{2m}\left\{p[p, x] + [p, x]p\right\}$$

NOW CALCULATE $[p, x]$ BY ACTING ON ARBITRARY $f(x)$

$$[p, x] f(x) = \left(-i\hbar \frac{\partial}{\partial x} x + x i\hbar \frac{\partial}{\partial x} \right) f$$

$$= -i\hbar \left\{ \frac{\partial x}{\partial x} f + \cancel{x \frac{\partial f}{\partial x}} - \cancel{x \frac{\partial f}{\partial x}} \right\}$$

$$= -i\hbar f(x)$$

$$\boxed{[p, x] = -i\hbar}$$

FUNDAMENTAL PROPERTY OF X AND p OPS.

HENCE

$$[H, x] = \frac{1}{2m} \left\{ p(-i\hbar) + (-i\hbar) p \right\}$$

$$= -i\hbar \frac{p}{m}$$

THUS

$$\boxed{\frac{d\langle x \rangle}{dt} = \frac{\langle p \rangle}{m}} \quad (\dagger)$$

AND POSITION IS NOT CONSERVED UNLESS IN SPECIAL STATE WITH $\langle p \rangle = 0$

NOTE : $(\dagger)$ IS QM ANALOGUE TO CLASSICAL

$$\dot{x} = p/m$$

FROM $(*)$ AND $(\dagger)$ SEE THAT CLASSICAL EQNS OF MOTION ARE OBEYED <u>ON THE AVERAGE</u> IN QM (EHRENFEST THM.)