### The physics of the cosmic microwave background





Acknowledgment: Anthony Challinor (University of Cambridge) for invaluable help in preparing these lectures.

**Disclaimers:** 

Time constraints necessitate brevity.

Lectures intended to give only a flavour of the physics.

Absolutely necessary to work through details (with the aid of a good textbook & the primary literature).

Recommended: Scott Dodelson, "Modern Cosmology" (2003)

#### Roadmap

Lecture 1: The physics of the cosmic microwave background.

Lecture 2: What have we learnt about the early universe?

### The Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

#### **Space-time and CMB Physics**

(NOT to scale)



# **Thermal History**

- CMB and matter plausibly produced during reheating at end of inflation
- CMB decouples around recombination, 300 kyr later
- Universe starts to reionize once first stars (?) form (somewhere in range z = 10-20) and 10% of CMB re-scatters



# Mapping the CMB

- WMAP3 internal-linear combination map (left) and BOOMERanG03 (right)
- Aim to answer in these lectures:
  - What is the physics behind these images?
  - What have we learned about cosmology from them?



## CMB spectrum and dipole anisotropy

- Microwave background almost perfect blackbody radiation
  - Temp. (COBE-FIRAS) 2.725 K
- Dipole anisotropy  $\Delta T/T = \beta \cos \theta$  implies solar-system barycenter has velocity  $v/c \equiv \beta = 0.00123$  relative to 'rest-frame' of CMB
- Variance of intrinsic fluctuations first detected by COBE-DMR:  $(\Delta T/T)_{\rm rms} = 16\mu {\rm K}$  smoothed on 7° scale



# Details of the thermal history

- Dominant element hydrogen recombines rapidly around  $z \approx 1000$ 
  - Prior to recombination, Thomson scattering efficient and mean free path  $1/(n_e\sigma_T)$  short cf. expansion time 1/H
  - Little chance of scattering after recombination  $\rightarrow$  photons free stream keeping imprint of conditions on last scattering surface
- Optical depth back to (conformal) time
   η for Thomson scattering:

 $\tau(\eta) = \int_{\eta}^{\eta_R} a n_e \sigma_T \, d\eta'$ 

- $e^{-\tau}$  is prob. of no scattering back to  $\eta$
- Visibility is probability of last scattering at η per dη:

visibility( $\eta$ ) =  $-\dot{\tau}e^{-\tau}$ 



#### Anisotropies and the power spectrum

Decompose temperature anisotropies in spherical harmonics

$$\Theta \equiv \Delta T(\hat{n})/T = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

- Under a rotation (*R*) of sky  $a_{lm} \rightarrow D^l_{mm'}(R)a_{lm'}$
- Demanding statistical isotropy requires, for 2-point function

$$\langle a_{lm}a^*_{l'm'}\rangle = D^l_{mM}D^{l'*}_{m'M'}\langle a_{lM}a^*_{l'M'}\rangle \quad \forall R$$

– Only possible (from unitarity  $D_{Mm}^{l*}D_{Mm'}^{l}=\delta_{mm'}$ ) if

$$\langle a_{lm}a^*_{l'm'}\rangle = C_l\delta_{ll'}\delta_{mm'}$$

- Symmetry restricts higher-order correlations also, but for *Gaussian* fluctuations all information in *power spectrum*  $C_l$
- Estimator for power spectrum  $\hat{C}_l = \sum_m |a_{lm}|^2/(2l+1)$  has mean  $C_l$  and cosmic variance

$$\operatorname{var}(\widehat{C}_l) = \frac{2}{2l+1}C_l^2$$

#### Measured power spectrum



## **Coupled Einstein-Boltzmann Equations**



### **Einstein Equation**

Relates the geometry to the energy:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

 $g_{\mu\nu}$  metric tensor: "machine" to transform coordinates into physical invariants.

 $R_{\mu\nu}$  Ricci tensor: depends on metric and its derivatives via Christoffel symbols.

R~ Ricci scalar: contraction of Ricci tensor,  $R\equiv g^{\mu\nu}R_{\mu\nu}$ 

 $G_{\mu\nu}$  Einstein tensor: describes geometry.

 $T_{\mu\nu}$  Energy-momentum tensor: describes energy content.

Given a metric, LHS: (i) compute Christoffel symbols (ii) form Ricci tensor (iii) contact to form Ricci scalar (iv) form Einstein tensor. Finally equate to RHS.

#### Perturbed FRW metric in CNG

The Conformal Newtonian Gauge describes the perturbed FRW metric.

$$ds^{2} = a^{2}(\eta) [(1+2\Psi)d\eta^{2} - \delta_{ij}(1-2\Phi)dx^{i}dx^{j}]$$

where  $\eta$  = conformal time. Metric perturbations described by scalar potentials  $\Psi$  (Newtonian potential) and  $\Phi$  (perturbation to spatial curvature).

Only describes scalar perturbations.

By the Decomposition Theorem, scalars, vectors and tensors evolve independently. Only scalars couple to matter.

### **Boltzmann Equation**

Want to describe distributions of photons and matter inhomogeneities. Photons and neutrinos are only fully described by distribution functions (DFs) in phase space. Energy-momentum tensor is expressed through integrals over momenta of the DFs.

$$rac{df}{dt} = C[f]$$
 RHS: all possible collision terms

The DF gives number of particles in (invariant) phase space volume  $d^3 \vec{x} d^3 \vec{p}$ :

$$dN = f(\vec{x}, \vec{p}, t)d^3\vec{x}d^3\vec{p}$$

Zeroth order: Bose-Einstein (bosons) and Fermi-Dirac (fermions) DFs.

Collision terms due to Thomson or Coulomb scattering need to be computed for all species except cold dark matter.

#### **Temperature anisotropies**

- On degree scales, scattering time short c.f. wavelength of fluctuations and (local!) temperature is uniform plus dipole:  $\Theta_0 + e \cdot v_b$
- Observed temperature anisotropy is snapshot of this at last scattering but modified by gravity:

$$[\Theta(\hat{n}) + \psi]_R = \underbrace{\Theta_0|_*}_{\text{temp.}} + \underbrace{\psi|_*}_{\text{gravity}} + \underbrace{e \cdot v_b|_*}_{\text{Doppler}} + \underbrace{\int_*^R (\dot{\psi} + \dot{\phi}) \, d\eta}_{\text{ISW}}$$

with line of sight  $\hat{n} = -e$ , and  $\Theta_0$  isotropic part of  $\Theta$ 

- Ignores anisotropic scattering, finite width of visibility function (i.e. last-scattering surface) and reionization
  - \* Will fix these omissions shortly

# Spatial to angular projection

• Consider angular projection at origin of potential  $\psi(x, \eta_*)$  over last-scattering surface; for a single Fourier component

$$egin{aligned} \psi(\widehat{m{n}}) &= \psi(\widehat{m{n}} \Delta \eta, \eta_*) & \Delta \eta \equiv \eta_R - \eta_* \ &= \psi(m{k}, \eta_*) \sum_{lm} 4\pi i^l j_l (k \Delta \eta) Y_{lm}(\widehat{m{n}}) Y_{lm}^*(\widehat{m{k}}) \ &\psi_{lm} \sim 4\pi \psi(m{k}, \eta_*) i^l j_l (k \Delta \eta) Y_{lm}^*(\widehat{m{k}}) \end{aligned}$$

•  $j_l(k \Delta \eta)$  peaks when  $k \Delta \eta \approx l$  but for given l considerable power from  $k > l/\Delta \eta$ also (wavefronts perpendicular to line of sight)





- CMB anisotropies at multipole *l* mostly sourced from fluctuations with linear wavenumber  $k \sim l/\Delta \eta$  where conformal distance to last scattering  $\approx 14$  Gpc

### CMB as a sound wave



Peebles & Yu (1970), Sunyaev & Zel'dovich (1970), Sachs and Wolfe (1968), Silk (1968)

# SHO analogy I

 $F_0$ 

Consider simple harmonic oscillator with mass m, force constant k driven by external force  $F_0$ .

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}$$

Assuming oscillator is initially at rest,

$$x = A\cos(\omega t) + \frac{F_0}{m\omega^2}$$
  
Peaks at  $t = \frac{n\pi}{\omega}$ .

unforced: peak heights equal forced: odd (even) peaks higher (lower) forcing disparity greater for lower Even peaks correspond to negative x

Figure from Dodelson, "Modern Cosmology"



# SHO analogy II

Cartoon gravitational instability:  $\ddot{\delta} + [Pressure - Gravity]\delta = 0$ Oscillator analogy:  $F_0$ 

 $\ddot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F$ 

F is force due to gravity,  $c_s^2$  is sound speed of entire photon-baryon fluid.

Add more baryons; sound speed (frequency) goes down.

Modes enter horizon and start to oscillate. See their phases frozen at recombination. Peaks are maximum amplitudes.

Figure from Dodelson, "Modern Cosmology"



# **Acoustic physics**

- Photon isotropic temperature  $\Theta_0$  and electron velocity  $v_b$  at last scattering depend on acoustic physics of pre-recombination plasma
- Large-scale approximation: ignore diffusion and slip between CMB and baryon bulk velocities (requires scattering rate  $\gg k$ )
  - Photon-baryon plasma behaves like perfect fluid responding to gravity (drives infall to wells), Hubble drag of baryons, gravitational redshifting and baryon pressure (resists infall):

$$\ddot{\Theta}_{0} + \underbrace{\frac{\mathcal{H}R}{1+R}\dot{\Theta}_{0}}_{\text{Hubble drag}} + \underbrace{\frac{1}{3(1+R)}k^{2}\Theta_{0}}_{\text{pressure}} = \underbrace{\ddot{\phi}}_{\text{redshift}} + \frac{\mathcal{H}R}{1+R}\dot{\phi} - \underbrace{\frac{1}{3}k^{2}\psi}_{\text{infall}}$$
where  $R \equiv 3\rho_{b}/(4\rho_{\gamma}) \propto a$ 

- WKB solutions of homogeneous equation:

 $(1+R)^{-1/4}\cos kr_s \quad , \quad (1+R)^{-1/4}\sin kr_s$ with sound horizon  $r_s \equiv \int_0^\eta \frac{d\eta'}{\sqrt{3(1+R)}}$  sound speed  $c_s = \frac{1}{\sqrt{3(1+R)}}$ 

# Gravitational potential and acoustic driving

- For adiabatic initial conditions (e.g. simple inflation models), no *relative* perturbations between number densities of species
  - Density perturbations of all species vanish on same hypersurface its curvature equals comoving curvature  $\mathcal{R}$  on super-Hubble scales
- Adiabatic driving term mimics  $\cos k\eta$ 
  - Oscillator is resonantly driven inside sound horizon whilst CDM sub-dominant
  - Potentials constant in matter domination then decay as DE dominates



#### Acoustic oscillations: adiabatic models

- $\delta_{\gamma}/4 \equiv \Theta_0$  starts out constant at  $-\psi(0)/2 \Rightarrow$  cosine oscillation  $\cos kr_s$  about equilibrium point  $-(1+R)\psi$ 
  - Modes with  $k \int_0^{\eta_*} c_s d\eta = n\pi$  are at extrema at last scattering  $\Rightarrow$  acoustic peaks in power spectrum
  - $v_b \approx v_\gamma$  follows from continuity equation ( $\pi/2$  out of phase with  $\Theta_0$  so Doppler effect 'fills in' zeroes of  $\Theta_0 + \psi$ )



#### Adiabatic anisotropy power spectrum

• Temperature power spectrum for scale-invariant curvature fluctuations



# **Tightly coupled solution**

Elegant solution first due to Sugiyama & Hu (1995):

$$\Theta_0(\eta) + \Phi(\eta) = [\Theta_0(0) + \Phi(0)] \cos(kr_s) + \frac{1}{\sqrt{3}} \int_0^{\eta} d\eta' [\Theta_0(\eta') + \Phi(\eta')] \sin[k(r_s(\eta) - r_s(\eta'))].$$

• Gets peak locations right and obtains odd/even height disparities fairly well.

- Neatly divides problem into
  - (i) calculation of external gravitational potentials generated by CDM.
  - (ii) effect of these potentials on the anisotropies.

• Clearly illustrates that cosine mode is the one excited by inflationary models.

## **Complications: photon diffusion**

- Photons diffuse out of dense regions damping inhomogeneities in ⊖<sub>0</sub> (and creating higher moments of ⊖)
  - In time  $d\eta$ , when mean-free path  $\ell = (an_e\sigma_T)^{-1} = 1/|\dot{\tau}|$ , photon random walks mean square distance  $\ell d\eta$
  - Defines a diffusion length by last scattering:

 $k_D^{-2} \sim \int_0^{\eta_*} |\dot{\tau}|^{-1} d\eta \approx 0.2 (\Omega_m h^2)^{-1/2} (\Omega_b h^2)^{-1} (a/a_*)^{5/2} \operatorname{Mpc}^2$ 

 Get exponential suppression of photons (and baryons)

 $\Theta_0 \propto e^{-k^2/k_D^2} \cos kr_s$ 

on scales below  $\sim 30 \, \text{Mpc}$  at last scattering

- Implies  $e^{-2l^2/l_D^2}$  damping tail in power spectrum



## **Integrated Sachs-Wolfe Effect**

- Linear  $\Theta_{\text{ISW}} \equiv \int (\dot{\phi} + \dot{\psi}) d\eta$  from late-time dark-energy domination and residual radiation at  $\eta_*$ ; non-linear small-scale effect from collapsing structures
  - In adiabatic models early ISW adds coherently with SW at first peak since  $\Theta_0 + \psi \sim -\psi/2$  same sign as  $\dot{\psi}$
  - Late-time effect is large scale (integrated effect ⇒ peak–trough cancellation suppresses small scales)
  - Late-time effect in dark-energy models produces positive correlation between large-scale CMB and LSS tracers for z < 2



# **Reionization: effect of rescattering**

- Lyman- $\alpha$  optical depth, as measured by quasar absorption spectra, rises rapidly around  $z \sim 6$  probing the end of the epoch of reionization
- CMB Thomson scatters off all (re-)ionized gas back to  $z_*$  with optical depth  $\tau = \int_{\eta_*}^{\eta_R} a n_e \sigma_T d\eta$ 
  - Produces a further low redshift peak in the visibility function (important for polarization – see later)



## **Reionization: effect of rescattering**

 CMB re-scatters off re-ionized gas; ignoring anisotropic (Doppler and quadrupole) scattering terms, locally at reionization have

 $\Theta(e) + \psi \to e^{-\tau} [\Theta(e) + \psi] + (1 - e^{-\tau})(\Theta_0 + \psi)$ 

- Outside horizon at reionization,  $\Theta(e) \approx \Theta_0$  and scattering has no effect
- Well inside horizon,  $\Theta_0 + \psi \approx 0$  and observed anisotropies

 $\Theta(\hat{n}) \to e^{-\tau} \Theta(\hat{n}) \quad \Rightarrow \quad C_l \to e^{-2\tau} C_l$ 



#### Gravitational waves

- Tensor metric perturbations  $ds^2 = a^2[d\eta^2 (\delta_{ij} + h_{ij})dx^i dx^j]$  with  $\delta^{ij}h_{ij} = 0$ 
  - Shear  $\propto \dot{h}_{ij}$  gives anisotropic redshifting  $\Rightarrow$

$$\Theta(\hat{n}) \approx -\frac{1}{2} \int d\eta \, \dot{h}_{ij} \hat{n}^i \hat{n}^j$$

- Only contributes on large scales since  $h_{ij}$  decays like  $a^{-1}$  after entering horizon



#### Isocurvature modes

- CDM isocurvature most physically motivated (perturb CDM relative to everything else)
- Starts off with  $\delta_{\gamma}(0) = \phi(0) = 0$  so matches onto  $\sin kr_s$  modes
- Temperature power spectrum for  $n_{iso} = 2$  entropy fluctuations (CDM isocurvature mode)



#### CMB polarization: Stokes parameters

• For plane wave along *z*, symmetric trace-free correlation tensor of electric field *E* defines (transverse) linear polarization tensor:



• Under right-handed rotation of x and y through  $\psi$  about propagation direction (z)

 $Q \pm iU \rightarrow (Q \pm iU)e^{\mp 2i\psi} \Rightarrow Q + iU$  is spin -2

## E and B modes

• Decomposition into *E* and *B* modes (use  $\theta$ ,  $-\phi$  basis to define *Q* and *U*)

 $\mathcal{P}_{ab}(\hat{n}) = \nabla_{\langle a} \nabla_{b \rangle} P_E + \epsilon^c{}_{(a} \nabla_{b)} \nabla_c P_B$  $\Rightarrow \quad Q + iU = \overline{\eth} \overline{\eth} (P_E - iP_B)$ 

- Spin-lowering operator:  $\bar{\eth}_s \eta = -\sin^{-s}\theta(\partial_{\theta} i\csc\partial_{\phi})(\sin^s\theta_s\eta)$
- Expand  $P_E$  and  $P_B$  in spherical harmonics, e.g.

$$P_E(\hat{n}) = \sum_{lm} \sqrt{\frac{(l-2)!}{(l+2)!}} E_{lm} Y_{lm}(\hat{n}) \quad \Rightarrow \quad (Q \pm iU)(\hat{n}) = \sum_{lm} (E_{lm} \mp iB_{lm})_{\mp 2} Y_{lm}(\hat{n})$$

- Spin-weight harmonics  ${}_{s}Y_{lm}$  provide orthonormal basis for spin-s functions
- Only three power spectra if parity respected in mean:  $C_l^E$ ,  $C_l^B$  and  $C_l^{TE}$

Pure  $E \mod e$ 

Pure  $B \mod e$ 





# CMB polarization: Thomson scattering



 Thomson scattering of radiation quadrupole produces linear polarization (dimensionless temperature units!)

$$d(Q \pm iU)(e) = \frac{3}{5}an_e\sigma_T d\eta \sum_{m} \pm 2Y_{2m}(e) \left(E_{2m} - \sqrt{\frac{1}{6}}\Theta_{2m}\right)$$

- Purely electric quadrupole (l = 2)
- In linear theory, generated Q + iU then conserved for free-streaming radiation
  - Suppressed by  $e^{-\tau}$  if further scattering at reionization

## CMB polarization: scalar perturbations

• Single plane wave of scalar perturbation has  $\Theta_{2m} \propto Y_{2m}^*(\hat{k}) \Rightarrow$  with  $\hat{k}$  along z,  $dQ \propto \sin^2 \theta$  and dU = 0



Plane-wave scalar quadrupole Electric quadrupole (m = 0) Pure *E* mode

- Linear scalar perturbations produce only E-mode polarization
- Mainly traces baryon velocity at recombination  $\Rightarrow$  peaks at troughs of  $\Delta T$

## CMB polarization: tensor perturbations

• For single +-polarized gravity wave with  $\hat{k}$  along z,  $\Theta_{2m} \propto \delta_{m2} + \delta_{m-2}$  so  $dQ \propto (1 + \cos^2 \theta) \cos 2\phi$  and  $dU \propto -\cos \theta \sin 2\phi$ 



Gravity waves produce both *E*- and *B*-mode polarization (with roughly equal power)

## Correlated polarization in real space

- On largest scales, infall into potential wells at last scattering generates e.g. tangential polarization around large-scale hot spots
- Sign of correlation scale-dependent inside horizon



#### Polarization from reionization

$$(Q \pm iU)(\hat{\mathbf{n}}) = -\frac{\sqrt{6}}{10} \int dD \frac{d\tau}{dD} e^{-\tau(D)} \times \sum_{m=-2}^{2} T_{2m}(D\hat{\mathbf{n}})_{\pm 2} Y_{2m}(\hat{\mathbf{n}})$$

## Large angle polarization from reionization

- Temperature quadrupole at reionization peaks around  $k(\eta_{re} \eta_*) \sim 2$ 
  - Re-scattering generates polarization on this linear scale  $\rightarrow$  projects to  $l \sim 2(\eta_0 \eta_{re})/(\eta_{re} \eta_*)$
  - Amplitude of polarization  $\propto$  optical depth through reionization  $\rightarrow$  best way to measure  $\tau$  with CMB



#### Scalar & tensor power spectra

• For scalar perturbations (left),  $\delta_{\gamma}$  oscillates  $\pi/2$  out of phase with  $v_{\gamma} \Rightarrow C_l^E$  peaks at minima of  $C_l^T$ 



## Parameters from CMB: matter & geometry

- Acoustic physics (dark energy and curvature negligible):
  - Peak locations depend on sound horizon  $r_s$  at last scattering
  - Damping scale  $1/k_D$  (roughly geometric mean of horizon and mean free path)
  - Both depend only on  $\Omega_b H_0^2$  and  $\Omega_m H_0^2$  for fixed  $T_{CMB}$
  - Peak heights depend on baryon loading  $(\Omega_b H_0^2)$  and gravitational driving  $(\Omega_m H_0^2)$ ; see shortly)  $\rightarrow r_s$  and  $k_D$  then calibrated standard rulers
- Main influence of geometry, dark energy and sub-eV massive neutrinos then through angular diameter distance to last scattering
  - $d_A$  accurately determined from angular size of standard rulers  $r_s$  and  $k_D$
  - Weak influence on large scales (where cosmic variance bad) through ISW

### Parameters from CMB: primordial power spectrum

- Scalar power spectrum C<sub>l</sub> essentially e<sup>-2</sup> 𝒫<sub>𝔅</sub>(k) at k ≈ l/d<sub>𝔅</sub> processed by acoustic physics
  - CMB probes scales  $5 \text{ Mpc} < k^{-1} < 5000 \text{ Mpc}$
- Tensor power spectra sensitive to  $e^{-2\tau}\mathcal{P}_h(k)$



# Degeneracies

- Some parameters not determined by linear T anisotropies alone:
  - Angular diameter test gives only  $d_A = d_A(\Omega_K, \Omega_{de}, w, ...)$  once matter densities determined from peak morphology
    - \* Disentangling dark energy and *K* relies on large-scale anisotropies, where cosmic variance large, or other datasets (e.g. Hubble, supernovae, shape of matter power spectrum or baryon oscillations)
  - Addition of gravity waves and renormalisation mimics reionization but can break with *polarization*



#### **Current temperature data**



- Sachs-Wolfe Plateau and late-time ISW effect
- Acoustic peaks at 'adiabatic' locations
- Damping tail/photon diffusion
- Weak gravitational lensing (see later)

#### **Current polarization data**



- Acoustic peaks at 'adiabatic' locations
- *E*-mode polarization and cross-correlation with  $\Delta T$
- Large-angle polarization from reionization

# Acoustic peak heights: baryon density

- Peak spacing fixed by  $r_s(\Omega_m h^2, \Omega_b h^2)$  and angular diameter distance  $d_A$ 
  - $\Theta_0$  oscillates with midpoint  $\approx -(1+R)\psi$  so  $\Theta_0 + \psi$  (S-W source) oscillates around  $-R\psi$
  - Increasing  $\Omega_b h^2$  (hence R) boosts compressional peaks (1, 3 etc. for adiabatic) and reduces  $r_s$
- Current constraints from CMB alone (weak priors):  $\Omega_b h^2 = 0.02273 \pm 0.00062$ (i.e. to 3%; Dunkley et al. 2008)
  - Should improve to sub-percent level with Planck data



# Acoustic peak heights: CDM density

- Increasing  $\Omega_c h^2$  reduces  $d_A$  and shifts matter-radiation equality to earlier times
  - Reduces resonant driving  $\ddot{\phi}$  for low-order peaks and reduces early-ISW contribution to 1st peak
- Current constraints from CMB alone (weak priors):  $\Omega_c h^2 = 0.1099 \pm 0.0062$  (i.e. to 6%; Dunkley et al. 2008)
  - Should improve to sub-percent level with Planck data



#### Peak locations: curvature & dark energy

- Mainly affect CMB through  $d_A$ ; small effects from ISW and mode quantisation for K > 0
  - CMB alone only well constrains  $d_A = 14.1 \pm 0.2 \text{ Gpc}$
  - $\Lambda = 0$ , closed models fit CMB alone but have very low *h*, high  $\Omega_m h$  cf. LSS, and don't fit ISW-LSS correlation (see later)
  - WMAP5 plus BAO gives  $\Omega_K = -0.005 \pm 0.006$  (w = -1) and  $\Omega_{\Lambda} = 0.734 \pm 0.017$



## Reionization

- WMAP5 *EE* large-angle correlation  $\Rightarrow \tau = 0.087 \pm 0.017$  (Dunkley et al. 2008)
  - Requires aggressive cleaning of polarized Galactic foregrounds (synchrotron and thermal dust emission)



## CMB spectrum: parameter dependences



Image from W. Hu

## Timeline



Image from D. Baumann