Problem Set 2 (2025) - Issued on 26th June 2025, Solutions Webinar on 2nd July 2025

1. Consider the diagram in Figure 1. Indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles.

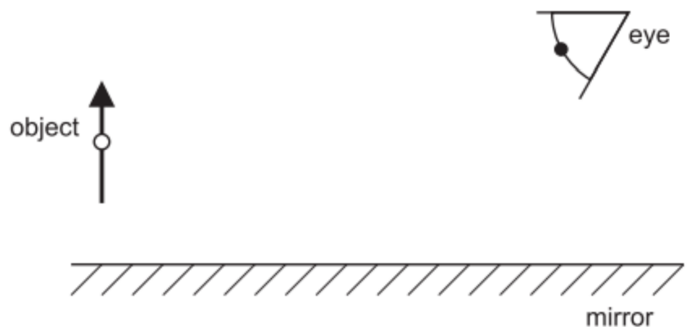

Figure 1: Indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles.

2. A parallel sided slab of medium B and refractive index n_B is sandwiched between two slabs of medium A of refractive index n_A. A beam of light passes from A through B and into A on the other side. If the beam is incident on B at an angle of θ to the normal, what is the angle to the normal of the light beam in A after it has left B?
   (a) $\cos^{-1}\left(\frac{n_A \sin\theta}{n_B}\right)$
   (b) θ
   (c) $\sin^{-1}\left(\frac{n_A^2 \sin\theta}{n_B^2}\right)$
   (d) $\sin^{-1}\left(\frac{n_A \sin\theta}{n_B}\right)$
   (e) $\frac{n_A}{n_B}\theta$

3. A parallel beam of monochromatic light, initially travelling in a direction above the horizontal, enters a region of atmosphere in which the refractive index increases steadily with height. Which of the graphs in Figure 2 represents the path of the beam of light?

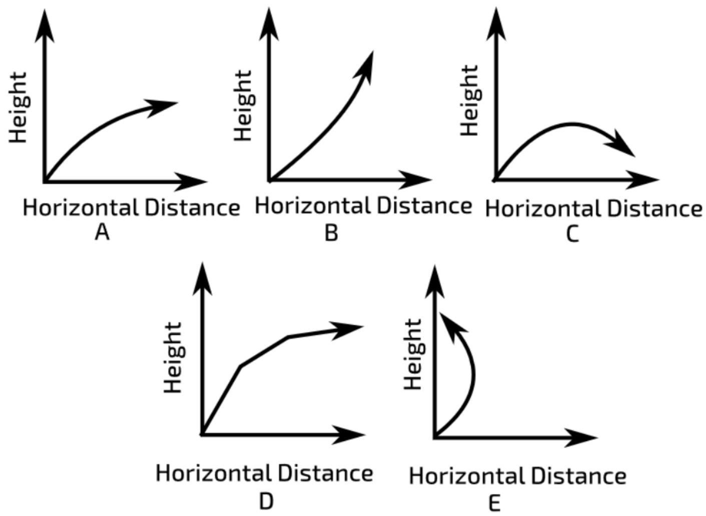

Figure 2: Which of these graphs represents the path of the beam of light?

4. A narrow beam of light is incident normally upon a thin slit. The light that passes through is spread out by diffraction. The thin slit is then immersed in a container of water. The beam of light is shone through the water and is again at normal incidence to the slit. The spread of the diffracted beam of light in water will be:

   (a) The same as in air
   (b) Diffraction will not occur in water
   (c) Less spread out than in air
   (d) More spread out than in air

5. Figure 4 shows two mirrors X and Y, and a solid object with white spots at P and Q.

   (a) An observer at A sees an image of P reflected in mirror Y. Mark R, the position of this image, and draw a ray from P to the observer at A.
   (b) In which mirror would an observer at A see an image of spot Q? Mark S, the position of this image.

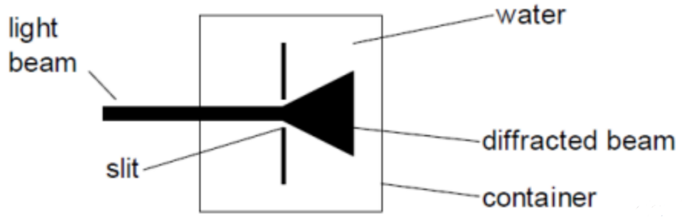

Figure 3: A narrow beam of light incident normally upon a thin slit in water.

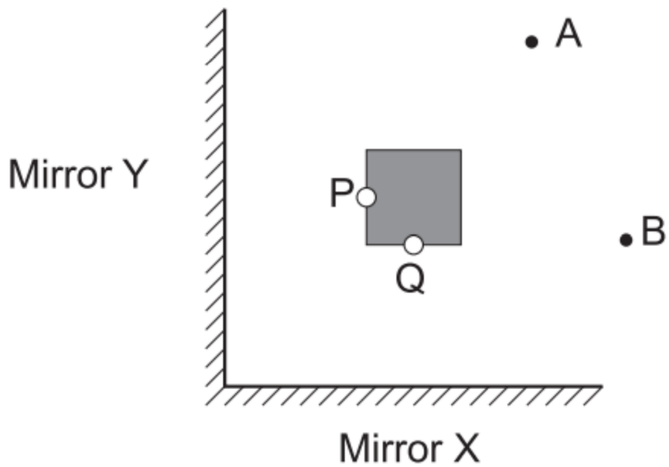

Figure 4: Two mirrors X and Y and a solid object with white spots at P and Q.

(c) An observer at B can see an image of P resulting from reflections at *both* mirrors. Draw a ray of light from P to B which enables this image to be seen.

6. A fisherman listens to the radio as he sits on the bank waiting for a fish to bite. The sound is also heard by the fish and the path of the sound waves entering the water is shown in Figure 5.

   (a) Describe what happens to the frequency, wavelength and speed of sound as it moves from air to water.

   (b) The fisherman's radio has two speakers, as shown in Figure 6.

   Sketch a diagram illustrating how destructive interference between sounds from the two speakers can occur when the radio is playing

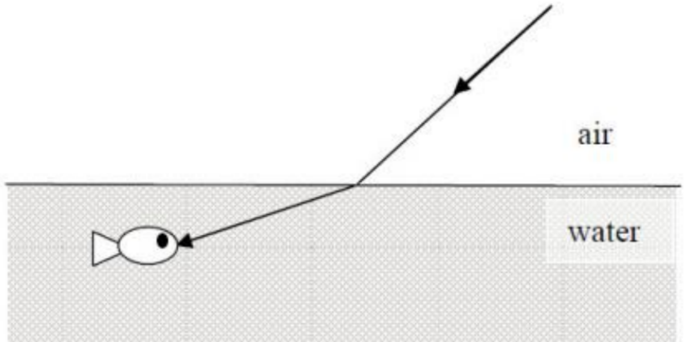

Figure 5: The path of the sound waves entering the water.

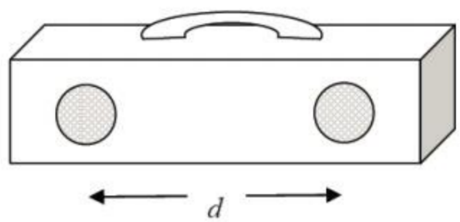

Figure 6: The speaker of the fisherman's radio.

a note of a single frequency, assuming that the waves from the two speakers start in phase.

(c) For a note of a single frequency and for a given separation of the two speakers, d, what must be the maximum wavelength λ and orientation of the radio for complete destructive interference to occur?

7. (a) Intensity decays as one moves further away from a source, due to the rays diverging. If I is the intensity and r is the distance from the source, then $I \propto r^n$ for what value of n?

(b) Rayleigh scattering is an effect that causes many optical phenomena. It is caused by the scattering of light by small particles, such as molecules that make up the air in the atmosphere.

If a beam of intensity I_0 and wavelength λ interacts with one of these particles, then the intensity of the light scattered at an angle θ is proportional to

$$I_0 \lambda^m r^n \alpha^6 (1 + \cos^2 \theta) \tag{1}$$

where r is the distance from the scattering particle and α is the diameter of the scattering particle. The relationship between the intensity of the scattered light (for a given wavelength) with the distance from the scattering particle is the same as for a point source. By considering the dimensions of the quantities involved, what is m to one significant figure?

8. A glass prism of refractive index $n = 1.40$ has a triangular cross section with two angles of 45°. The prism floats on some mercury with its largest side of length $l = 45.0\,\text{cm}$ facing downwards and a vertical depth of $h = 2.50\,\text{cm}$ submerged.

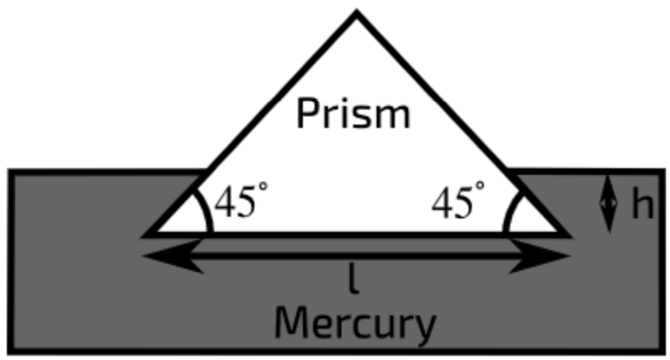

Figure 7: A triangular glass prism floating on some mercury.

   (a) A monochromatic beam of light, entering the glass parallel to the mercury surface, internally reflects off the bottom face of the prism due to the presence of the mercury. What is the maximum height of the incident beam above the mercury surface such that the beam can leave on the other side of the prism, parallel to the mercury surface?

   (b) The prism is then placed on top of a different, clear fluid of the same density and floats. What is the maximum refractive index of the fluid that will allow the light to travel along the same path as in part (a)?

9. On roads, devices known as cat's eyes are used to reflect light from a car's headlights back towards the driver. These are loosely based on how light that enters a cat's eye will be reflected back out in a similar direction, so the eye will often seem to glow at night.

   One type of cat's eye is created using a sphere of glass, with a curved mirror over half of its surface. Light entering the sphere is reflected off the mirror and exits the sphere travelling in the exact opposite direction to its direction of travel before entering the sphere (that is, at the same angle to the horizontal).

(a) A beam of light is incident on the surface of the sphere at an angle of $\theta_i = 4.58°$ to the normal of the sphere at that point. If the refractive index is $n = 1.54$, what is the angle through which the incident beam deviates as it is refracted at this first surface? This is the angle between its original direction and its new direction.

(b) Consider an idealised version of the cat's eye, whereby the entire sphere has a refractive index n. The deflection of the beam inside the sphere will depend on this refractive index. θ_i is the angle of incidence of the beam on the sphere and θ_r is the angle of refraction as the beam enters the sphere. What is the total deflection of the beam once it has emerged from the sphere, assuming it only reflects from the mirror once?

(c) Assuming that θ_i and θ_r are small so that the approximation $\sin\theta \approx \theta$ holds, what refractive index would be needed for the beam that has left the sphere to be moving in exactly the opposite direction to the beam before entering the sphere?

Hints to Workshop Session 4

1. Use a ruler to draw a ray diagram – what type of image is formed in a mirror?

2. Find the refraction angle as the light beam leaves the second surface (going from B to A). Draw a ray diagram and label refractive indices and angles. Use Snell's law and the alternate angle rule.

3. Make sure you pay attention to the fact that the refractive index increases with height. Think about the atmosphere consisting of many thin layers of different refractive indices, with each layer having a refractive index slightly greater than the one below. Draw a diagram showing refraction through one layer, and then consider multiple layers.

4. Consider how the refractive index of water changes the refraction – and therefore diffraction – of light through the slit.

5. Consider where reflection occurs when a ray travels between two points, and where the observer sees the image. Label the diagram carefully.

6. (a) Which property of waves remains invariant as waves travel between media? What does the path of the sound waves tell you about the waves' speed?

   (b) If the waves start in phase, how they can end in antiphase and thus result in destructive interference?

   (c) Your diagram in part (b) should help you here.

7. (a) Does intensity increase or decrease with distance? Can you remember what kind of law it follows? Other examples of this kind of law include Newton's law of gravitation and Coulomb's law.

   (b) Find the dependence of intensity on wavelength using dimensional analysis by using n from part (a). What are the units? Does it matter if you don't know the units for intensity?

8. (a) Add rays to the diagram and label known and unknown values. How do we know that the angle of incidence as the light enters the prism is $45°$? Use Snell's law to find the angle of refraction. Can you spot two similar triangles?

   (b) You want to find a limiting case for total internal reflection. Can you think of or derive an equation linking θ, n_{liquid} and n_{prism}?

9. (a) Draw a ray diagram and use Snell's law. Note that the question does not ask for the angle of refraction but the angle of deviation. How are the angles of incidence, refraction and deviation related?

   (b) Draw a new ray diagram. Do you already know multiple angles due to the symmetry of the problem? Calculate each deflection separately before adding them together.

   (c) If the beam is moving in exactly the opposite direction, then its total deflection (as you calculated in the previous part of the question) is $180°$. Use Snell's law and the small angle approximation.