Problem Set 4 (2025) - Issued on 25th September 2024, Solutions Webinar on 1st October 2025

1. A smooth marble is initially at rest at the top of a much larger smooth hemisphere of radius r. The marble is given a slight nudge and begins to slide down the hemisphere.

   (a) At what angle from the vertical will the marble leave the surface of the hemisphere?

   (b) How far away from the base will the marble land?

2. Consider a toy car going around a loop-the-loop. If the car is going too slowly around the loop-the-loop, at some point it will fall off.

   (a) If the car started at rest on a downwards ramp which was initially at the same height as the loop, would the car make it around safely? Explain why.

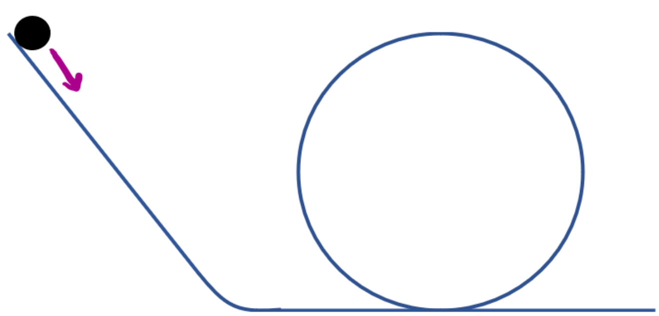

Figure 1: A car (black circle) at rest at the top of a downwards ramp which is the same height as the loop-the-loop.

   (b) Bob wants to find out if it's possible to do the loop-the-loop with a real car. He has built a loop which has a 6 m radius, and his car will approach the loop driving along a flat runway. What is the minimum speed, in mph, that Bob needs to drive at in order to perform the loop-the-loop successfully?

(c) Is there any reason why Bob shouldn't go much faster than this minimum speed?

3. A velodrome allows cyclists to travel at high speed around tight corners since the track is banked at a steep angle.

   (a) By drawing a force diagram, show why this banking is necessary. Determine an expression for the maximum speed a cyclist can travel at as a function of the radius r of the corner and the coefficient of dynamic friction μ.

   (b) Determine the value of μ required for a bike to travel at $80\,\text{kmh}^{-1}$ around corners of radius $25\,\text{m}$ with a maximum banking angle of $42°$.

4. Racing cars have spoilers which direct the air flow over the car upwards.

   (a) Explain why this makes the cars 'heavier' with reference to at least one of Newton's laws of motion.

   (b) A student suggests that this is a silly design feature, as the heavier an object is the slower it will go. What is the physics behind this argument, and is it correct?

   (c) Assume that the downforce created from the spoilers is proportional to v^2, where v is the velocity of the car. If it is possible for a racing car of mass 800 kg to drive on the roof of a tunnel provided it is travelling faster than 150 mph, determine the value of the constant of proportionality.

5. In Hertford, Hampshire and Hereford, hurricanes hardly happen. However, if they were to happen, would they likely rotate clockwise, anticlockwise or have no preference?

6. For an object undergoing simple harmonic motion, it is possible to express its velocity as a function of time (as it is simply the time derivative of its displacement). Show that an objecting undergoing simple harmonic motion has a velocity as a function of displacement is given by

$$v(x) = \pm\omega\sqrt{A^2 - x^2} \tag{1}$$

   where all the symbols have their usual meanings.

7. Imagine that a tunnel is constructed straight through the centre of the Earth. If a person were to fall into the tunnel, would they arrive at the other end? Describe the motion of the person and *either* explain why the person would not reach the other end *or* calculate the time taken for the person to travel from one end of the Earth to the other.

8. In The A-Team film, Hannibal and his team find themselves plummeting towards the Earth in a tank with only one of its three parachutes attached. This would not be a soft landing! However, there is a lake about half a

mile away from their landing spot. The team attempt to ‘fly the tank’ to the lake by firing shells horizontally. This question will examine whether this is pure Hollywood or based in sound physics.

For the team to be successful, how high up must they be when the execute this plan? You may ignore the effects of air resistance in the horizontal direction.

The following data may be useful:

- Projectile mass: 10 kg
- Muzzle velocity: $1750\,\mathrm{ms}^{-1}$
- Time between shots: 3.5 s
- Tank mass: 22 000 kg
- Terminal velocity: 33 mph

Hints to Workshop Session 4

1. (a) What do you know about the contact force at the point of interest?
   (b) Have you resolved your initial velocity into useful components? The marble in a fishbowl question from Workbook 1 may be a useful guide.
2. (a) What is the velocity of the car when it gets back to its starting height? When will the car have its lowest speed during the loop? Can you find an expression for the necessary minimum speed to keep going around the loop?
   (b) This question is not actually about circular motion – why do the normal equation of circular motion not apply in this scenario? Why is the speed at the bottom of the ramp different to that at the top? It may help to model the car as a marble.
   (c) What are the forces a person experiences if they change direction suddenly? What could happen to their body if these forces were too large? Think about astronauts being trained in a human ‘centrifuge’.
3. (a) Have you drawn a clear diagram showing all the forces acting on the cyclist? In which direction must the centripetal force act? Is this a sensible direction to resolve the forces?
   (b) Rearrange your answer for (a) to solve for μ.
4. (a) Newton’s third law is important here.
   (b) Although there is inertial mass and gravitational mass (which just so happen to be the same thing), does the weight of the car actually increase?

(c) What is the relationship between the weight of the car and the 'down-force'?

5. You should piece together the physics behind this phenomenon. Think about:

    - Most weather patterns are driven by the Sun.
    - What is the significance of isobars on a weather forecast?
    - Where do hurricanes usually occur and why?
    - What do the above questions have to do with circular motion?
    - What is the Coriolis effect?

6. You should be able to derive the fact that $v(t) = -A\omega \sin(\omega t)$. There is a useful trigonometric identity which will help to get rid of the sin and cos terms, if you square them.

7. What happens to the force due to gravity as the person approaches the centre of the Earth? As $r \to 0$, does the force not become infinite? What effect does this have on their velocity? How fast will they be travelling when they get to the centre of the Earth?

    It turns out that we can ignore all the mass at a greater radius from the centre of the Earth than the person is at any given point. This is because the gravitational pull from all of the material contained within this ring exactly cancels out. As the person falls towards the centre of the Earth, there is some mass above them, which is now pulling them upwards. But this exactly cancels out the pull of the other mass outside radius r.

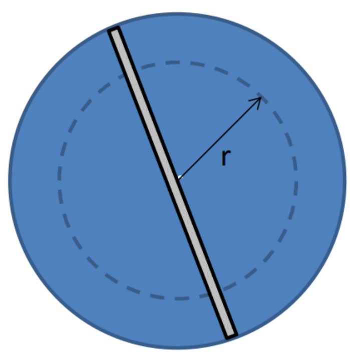

Figure 2: If the person is at radius r, then the gravitational pull from all the mass of the Earth contained outside of this radius exactly cancels out.

In other words, it is only the mass that is contained within a sphere of radius equal to the person's displacement from the centre of mass that contributes to the force of gravity. This means that you only have to consider the mass within the person's radius.

Recall that simple harmonic motion occurs if the acceleration of an object is proportional to, and in the opposite direction to, the object's displacement about the equilibrium position.

8. Make as many simplifying assumptions as you can. How important is it that the mass of the tank will decrease?

   Can you smooth out the force? Instead of having many impulses every 3.5 s, consider finding an average continuous force.

   If you can find a constant acceleration then regular SUVAT equations can be applied.