Study Focus 1 Problems - Used in Webinar on the 11th June 2025

It has often been said that "mathematics is the language of physics". In order to do well in physics, one must therefore become a good *translator*.

Problem-solving hints and tips

Many of the problems in the PAT are short and immediately obviously very mathematical. However, some are not and you will need to "decode" the question to turn the English and the physics into a mathematical problem that you can solve.

- Turn words into symbols. This is the most important point. Replace words with standard symbols and use subscripts to differentiate between objects. So "the speed of the car" becomes v_c and "the weight of the passenger" becomes w_p.
- Look for 'numerical words' like "twice" or "halves". If x is twice y then $x = 2y$.
- Look for words that imply an equality relationship like "same as" or "as [long/cold/big] as".
- Look for information hidden in the question:
  - Bear in mind conservation laws: if you know the total momentum at the start of the question, you know the total momentum at the end. The same goes for mass-energy and charge.
  - If you're dealing with particles, there's a very good chance that the charge on the particle is $1e$ or $-1e$. If it's an alpha particle or a beta particle then you know the charge is $2e$ or $-1e$ respectively. If you know the atomic number of a nucleus then you also know its charge.

  - Remember that some units are equivalent to each other. If you know an acceleration in $\mathrm{ms^{-2}}$, we also know it in $\mathrm{N\,kg^{-1}}$. A count rate in $\mathrm{s^{-1}}$ can also be in Bq or Hz. An electric field strength in $\mathrm{Vm^{-1}}$ (or if you know the voltage across a known distance) means you also know the field strength in $\mathrm{NC^{-1}}$.
  - This is especially important for projectiles/suvat questions. If a question takes place on Earth then $g = 9.81\,\mathrm{m\,s^{-2}}$. If something is "dropped" then this means that its initial speed is $0\,\mathrm{m\,s^{-1}}$. If it has reached maximum height then its vertical speed is $0\,\mathrm{m\,s^{-1}}$.

- Watch out for superfluous or "decoy" data in the question. You might not need to know the mass of an object, or the current in a circuit to answer the question. Imagine what would happen if the value became very large or very small: would it actually make a difference? Don't assume that every number in a question has to be used.

- Subsequent parts of a question may ask what effect changing one of the variables has on a previously obtained result. For example: "What would be the effect of doubling the resistance?". These questions can nearly always be solved just by looking at the factors involved. Only very rarely will you need to completely recalculate.

- Look at the units. If you're trying to find a "flow rate" then the units will most likely be $\mathrm{kg\,s^{-1}}$ or $\mathrm{m^3s^{-1}}$. Can you get to one of these with the information you have?

Some typical questions

Let's try a few questions together to see how we might decode them.

Example 1

A jar contains buttons of four different colours. There are twice as many yellow as green, twice as many red as yellow, and twice as many blue as red. What is the probability of taking from the jar:

1. a blue button;
2. a red button;
3. a yellow button;
4. a green button?

You may assume that you are only taking one button at a time and replacing it in the jar before selecting the next one.

Example 2

A gun is designed that can launch a projectile of mass 10 kg at a speed of 200 ms^{-1}. The gun is placed close to a straight, horizontal railway line and aligned such that the projectile will land further down the line. A small rail car of mass 200 kg and travelling at a speed of 100 ms^{-1} passes the gun just as it is fired. Assuming the gun and the car are at the same level, at what angle upwards must the projectile be fired in order that it lands in the rail car?

Example 3

The planet Pluto (radius 1180 km) is populated by three species of purple caterpillar. Studies have established the following facts:

1. A line of 5 mauve caterpillars is as long as a line of 7 violet caterpillars.
2. A line of 3 lavender caterpillars and 1 mauve caterpillar is as long as a line of 8 violet caterpillars.
3. A line of 5 lavender caterpillars, 5 mauve caterpillars and 2 violet caterpillars is 1 m long in total.
4. A lavender caterpillar takes 10 s to crawl the length of a violet caterpillar.
5. Violet and mauve caterpillars both crawl twice as fast as lavender caterpillars.

How long would it take a mauve caterpillar to crawl around the equator of Pluto?

Study Focus 1: Problems

1. The drag force F on a sphere is related to the radius of the sphere, r, the velocity of the sphere, v, and the coefficient of viscosity of the fluid the drop is falling through, η, by the formula

$$F = kr^x \eta^y v^z$$

   where k is a dimensionless constant, and x, y, and z are integers. By considering the units of the equation, work out the values of x, y, and z. (The coefficient of viscosity has units of kg $m^{-1}s^{-1}$.)

2. A radioactive sample contains two different isotopes, A and B. A has a half-life of 3 days, and B has a half-life of 6 days. Initially in the sample there are twice as many atoms of A as of B. At what time will the ratio of the number of atoms of A to B be reversed?

3. A snooker ball must be 5.175 cm in diameter to within an uncertainty of ± 0.127 mm. The Earth is 6371 km in radius and its highest mountain above sea level, Mount Everest, is 8848 m. Which is smoother, a snooker ball or the Earth? [Note: do we know everything to the same level of accuracy?]

Hints for Study Focus 1

1. Essentially, this problem boils down to solving the equation:

$$\mathrm{kg\,m\,s^{-2}} = [\mathrm{m}]^x[\mathrm{kg\,m^{-1}s^{-1}}]^y[\mathrm{m\,s^{-1}}]^z$$

   Remember that $(ab)^x = a^x b^x$ and note that there is only one kilogram term on either side of the equals sign.

2. There are two factors of two in the question: there are twice as many atoms of A than B, so $N_A = 2N_B$ and the half-life of B is twice that of A. Use $N = N_0 e^{-\lambda t}$ and remember that you are looking for the situation to be reversed, i.e. $N_A = \frac{N_B}{2}$.

3. Think about percentage error when approaching this question. Make sure you convert between millimetres, centimetres, metres and kilometres as there are a lot of different units in this question.

Solutions to Example Problems

Example 1

We will begin by assigning variables. The number of buttons will be represented by n with a subscript representing the colour of the buttons. The total number of buttons will be represented by N, such that $N = n_y + n_g + n_r + n_b$. We can now break the information in the question down piece-by-piece. "There are twice as many yellow as green" means that n_y is twice n_g or rather $n_y = 2n_g$. The same goes for "twice as many red as yellow": $n_r = 2n_y$ and "twice as many blue as red": $n_b = 2n_r$.

This gives us:

$$n_y = 2n_g \tag{1}$$
$$n_r = 2n_y \tag{2}$$
$$n_b = 2n_r. \tag{3}$$

If we assume that we have one green button then this means that we have two yellow buttons. If we have two yellow buttons, then this means that we have four red buttons. If we have four red buttons, then this means that we have eight blue buttons. The ratio of green : yellow : red : blue is 1:2:4:8.

The fact that we assumed we have one green button is unimportant: we could have picked any colour and chosen any value for it. The key thing is the ratios between the numbers of coloured buttons. For example, let's assume instead that there are 1000 yellow buttons. There are then 500 green buttons (Equation 1), 2000 red buttons (Equation 2) and 4000 blue buttons (Equation 3). The ratio of green : yellow : red : blue is then 500 : 1000 : 2000 : 4000, which is the same as above. Don't be afraid to try to put in numbers and see what happens.

Once we have the ratio of buttons, we have the probabilities:

- Blue is $\frac{8}{1+2+4+8} = \frac{8}{15}$,
- Red is $\frac{4}{1+2+4+8} = \frac{4}{15}$,
- Yellow is $\frac{2}{1+2+4+8} = \frac{2}{15}$ and
- Green is $\frac{1}{1+2+4+8} = \frac{1}{15}$.

Example 2

As mentioned before, for projectile / "suvat" questions it is important to look for information that is hidden in the question. For example, the question will almost never remind you that there is a vertical acceleration of $a = g = -9.81\,\mathrm{ms}^{-2}$. "Passes the gun just as it's fired" tells us that we can use the same frame-of-reference, the same t, for both. "Assuming that the gun and the car are at the same level" tells you that you don't have to worry about any vertical displacement or offset between the two objects. Also, given that this exists in the world of projectiles, in which objects are in free-fall, the masses of the objects are irrelevant. Their inclusion in the question is a red herring!

It is often useful to draw a diagram so let's start there:

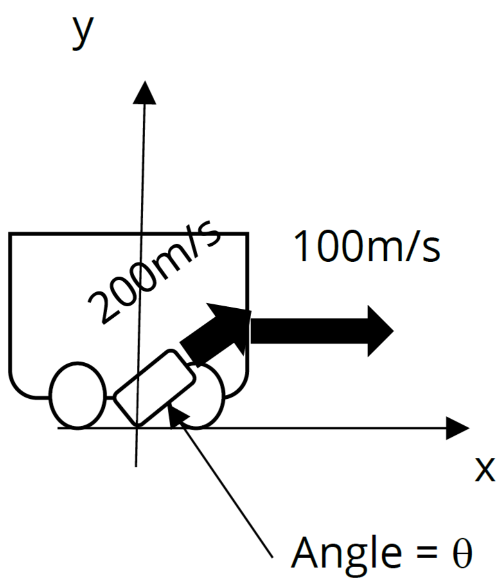

Figure 1: Example 2

There are two points at which the projectile and the car are in the same position. One is at $t = 0\,\mathrm{s}$ when the projectile is launched, and the second is at $t = t\,\mathrm{s}$ when the projectile lands in the car. At this point they therefore have the same horizontal displacement, s. The acceleration of the rail car is zero, both horizontally and vertically, and the acceleration of the projectile, if we take upward to be positive, is $a_p = -9.81\,\mathrm{ms}^{-2}$. Note how we've managed to "pull" a lot of information out of the question that we weren't explicitly given.

The horizontal speed of the projectile is $200\cos\theta\,\mathrm{ms}^{-1}$ (if we take θ to be the angle above the horizontal) and the horizontal speed of the car is $100\,\mathrm{ms}^{-1}$. Given that they must both travel the same distance in the same time for one to land in the other:

$$
\begin{aligned}
s_p &= s_c && (4)\\
(200\cos\theta)t &= 100t && (5)\\
\implies 200\cos\theta &= 100 && (6)\\
\implies \cos\theta &= \frac{100}{200} && (7)\\
\implies \theta &= \cos^{-1}\frac{100}{200} && (8)\\
\therefore \theta &= 60^\circ && (9)
\end{aligned}
$$

Example 3

To answer this question we need to know two things: the distance around Pluto's equator and the crawling speed of a mauve caterpillar. Since we are provided with the radius in the question, we can calculate the distance from the formula $2\pi r$. The crawling speed of a mauve caterpillar, though, is considerably more difficult to find.

The first thing to do is to replace as many words with symbols as possible. We could use 'm' to represent the length of a mauve caterpillar, 'v' to represent the length of a violet caterpillar, and 'l' to represent the length of a lavender caterpillar, but then what symbol would we use to represent the crawling speed of each? It will be easier to use standard symbols for quantities, and use subscripts to indicate which caterpillar we are writing about. Thus, the length of a mauve caterpillar will be l_m and the crawling speed of a lavender caterpillar will be v_l.

Our five pieces of information then become:

1. $5l_m$ is as long as $7l_v$
2. $3l_l + l_m$ is as long as $8l_v$
3. $5l_l + 5l_m + 2l_v$ is 1 m long in total.
4. A lavender caterpillar takes 10 s to crawl l_v.
5. v_v and v_m are twice v_l.

Here the "and"s in the question have become addition and the number of caterpillars of each colour have become simple multipliers. There are, however, still a lot of words left over here, so let's turn our lines of text into "proper" equations.

1. $5l_m = 7l_v$
2. $3l_l + l_m = 8l_v$
3. $5l_l + 5l_m + 2l_v = 1$

4. $v_l = \frac{l_v}{10}$

5. $v_v = 2v_l$ and $v_m = 2v_l$.

Where we are told something is "as long as" that is the same as saying they are equal. "Twice" means a factor of two. The fourth point, which – unlike the other four points – contained information about time taken, has now become about speed and distance, to make it more similar to the other four points.

Given that we are trying to find the speed of a mauve caterpillar, it makes sense to start with the second half of the fifth point, $v_m = 2v_l$ and combine it with the fourth point:

$$v_m = 2v_l \tag{10}$$
$$= 2\frac{l_v}{10} \tag{11}$$

So in order to answer the question we just need to find l_v, the length of a violet caterpillar. The first three points all contain a term in l_v:

1. $5l_m = 7l_v$

2. $3l_l + l_m = 8l_v$

3. $5l_l + 5l_m + 2l_v = 1$

We then rearrange these so that they're all in the same order and look similar to each other:

$$0l_l + 5l_m - 7l_v = 0 \tag{12}$$
$$3l_l + l_m - 8l_v = 0 \tag{13}$$
$$5l_l + 5l_m + 2l_v = 1 \tag{14}$$

The $0l_l$ term is obviously unnecessary, but worth including in that it makes the similarities between the three equations more obvious.

We now have a set of simultaneous equations that we can solve in the usual way. Given that we already have one equation without a term in l_l it makes sense to eliminate that term from the combination of the other two equations.

Multiplying equation 13 by 5/3 gives

$$5l_l + \frac{5}{3}l_m - \frac{40}{3}l_v = 0.$$

Subtracting from Equation 14 gives us that

$$\frac{10}{3}l_m = 1 - \frac{46}{3}l_v$$

or more simply,

$$l_m = \frac{3}{10} - \frac{23}{5}l_v.$$

You should not be put off if, in your solution, you encounter some more unusual fractions than you might expect in other exams.

Using this information, and the very first piece of information we were given, $5l_m = 7l_v$ we then find

$$5\left(\frac{3}{10} - \frac{23}{5}l_v\right) = 7l_v \tag{15}$$

$$\frac{3}{2} - 23l_v = 7l_v \tag{16}$$

$$\frac{3}{2} = 30l_v \tag{17}$$

$$\therefore l_v = 0.05\,\mathrm{m} \tag{18}$$

Remembering that $v_m = \frac{2l_v}{10}$ we now know that the speed of the mauve caterpillar is $0.01\,\mathrm{ms}^{-1}$.

Now that we have v_m, the speed of a mauve caterpillar, we can now find the answer. To find the circumference of Pluto we need to calculate $C = 2\pi r$, where r is given to us in the question as $r = 1180\,\mathrm{km}$. It would be very time-consuming to calculate a really accurate value of Pluto's circumference, but if we use the standard physicist's approximation of $\pi \approx 3$ then this becomes a little easier:

$$C = 2 \times 3 \times 1200 \tag{19}$$

$$= 7200\,\mathrm{km}. \tag{20}$$

Note that we've approximated 1180 km as 1200 km, but we also "under-estimated" the value of π so we've got a reasonable answer.

$$t = \frac{7200 \times 10^3}{1 \times 10^{-2}} \tag{21}$$

$$= 7200 \times 10^5 \tag{22}$$

$$= 7.2 \times 10^8\mathrm{s} \tag{23}$$

If we were using a calculator we'd get $t = 7.4 \times 10^8$s, which is less than 3% different.

Numerical answers can be found below.

1. 31.5Ω; 2. $\sqrt{\frac{17rg}{3}}$; 3. 400 N; 7. AB is $R/3$, AC is $R/2$, AD is $5R/6$.