Study Focus 2 Problems - Used in Webinar on the 25th June 2025

Warm Up Problems

1. Imagine that you have a height h and are standing at a distance d from a mirror, looking at your own reflection. In order to be able to see a full-length view of yourself, the minimum size of the plane mirror must be:

   (a) $h/4$
   (b) $h/2$
   (c) $3h/4$
   (d) h
   (e) Depends on the exact value of d

2. Zoe wishes to 'spear' a fish with a laser – that is, she wants to shine a laser onto a lake and have the light beam hit a fish below the surface of the water. Should she aim the laser beam above, below, or directly at the observed fish to make a direct hit?

Introductory Problems

3. A triangular glass prism sits on a table pointing upwards. A beam of coloured light is directed horizontally near the top of the prism, as shown in Figure 1. What happens to the light beam at the prism?

   (a) It is bent upwards
   (b) It is bent downwards
   (c) It continues horizontally
   (d) It depends on the colour of the light

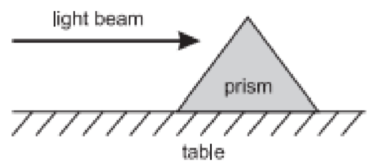

Figure 1: A beam of coloured light directed horizontally towards the top of a triangular glass prism.

4. A beam of light is incident from a vacuum onto a medium at an angle θ to the normal of the boundary. The refracted and partially reflected beams happen to form a right angle. Find an expression for the refractive index of the medium.

Further Problems

5. This question concerns total internal reflection, optical fibres, and refraction. You may assume that the refractive index of glass is larger than that of water, and that the refractive index of water is larger than that of air.

   (a) Explain what is meant by the phrases *total internal reflection* and *critical angle*. (You are encouraged to use a diagram to explain your answer.)

   (b) Derive an equation relating the critical angle and the refractive indices of two materials, n_1 and n_2, where $n_2 < n_1$.

   (c) An optical fibre is usually made of two materials, a core and a cladding, as shown in Figure 2 (not drawn to scale).

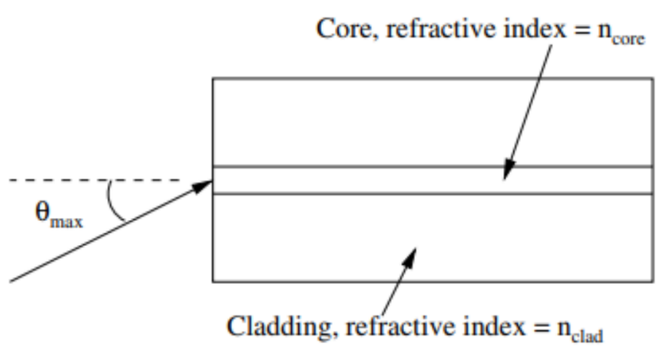

Figure 2: A diagram of an optical fibre.

Light may only be transmitted along the fibre if the incident angle of the light is less than a maximum angle θ_{max}. By using your equation from above and Snell's Law, or otherwise, derive an expression for θ_{max} in terms of the core and cladding refractive indices only.

6. In an optical fibre, light can travel directly down the middle of the fibre. Alternatively, a *meridional ray* is one which bounces off the walls of the fibre yet stays in a single plane. The minimum angle a ray can bounce at is controlled by the critical angle. For a glass fibre with a core index of 1.500, a cladding index of 1.496 and length 1 km:

   (a) Calculate the maximum path length for the meridional ray.

   (b) Hence calculate the time difference for this ray and a ray which passes straight through.

   (c) If square (in time) pulses of light are used to send information down the fibre, calculate the maximum rate at which information can be sent.

7. In a particle physics experiment, light from a particle detector is to be collected and concentrated by reflecting it between a pair of plane mirrors with angle 2α between them, as shown in Figure 3. A faint parallel beam of light consisting of rays parallel to the central axis is to be narrowed down by reflection off the mirrors, as shown by the single ray illustrated, for which angle $a = \alpha$.

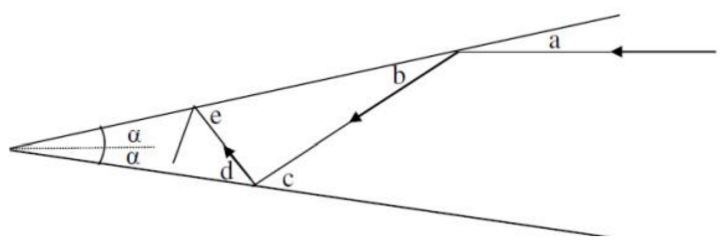

Figure 3: A parallel beam of light being reflected between a pair of plane mirrors.

   (a) Determine angles b, c, d and e in terms of angle α.

   (b) Explain what happens after several reflections of the light down the mirror funnel.

   (c) If $\alpha = 10°$, what is the total number of reflections between the mirrors that will be made by a beam of light entering parallel to the axis of symmetry as shown?

   (d) If the mirrors are replaced by an internally silvered circular cone whose cross-section is the same as that shown above, why will this not make any difference to the calculation given above for the plane angled mirrors with a beam of light parallel to the axis?

(e) An ear trumpet was a device that was used to collect sound and focus it into the ear. It was a cone about 0.5 m long with an angle 2α of about $30°$. The sound passing into the device would typically have a frequency of 400 Hz and a speed of $330\,\mathrm{ms}^{-1}$. Why is the model above that we have used for light not valid for an ear trumpet used to collect sound?

Hints to Study Focus 3

Warm Up Problems

1. Draw a ray diagram. How can you see both your head and your feet in a mirror? Remember the law of reflection.

2. First think about where Zoe would need to aim if she were throwing a physical spear into the water and wanted to hit the fish directly. How does shining a laser beam affect the physics involved?

Introductory Problems

3. Read the question carefully – it is a beam of coloured light, for example red light or green light. Remember the dispersion of white light by a prism. What happens to the white light and why? Does it disperse by a lot or only by a little? Draw a ray diagram.

4. Draw a digram showing reflection and refraction at the surface. Label angles (both known and unknown). Use Snell's law and remember that $\sin(90° - \theta) = \cos\theta$.

Further Problems

5. (a) Draw a clear diagram to illustrate total internal reflection and critical angle with labels.

   (b) For the derivation, start by using Snell's law. What is the angle of refraction for light if the angle of incidence is equal to the critical angle? Optical fibres contain a core and cladding where $n_{core} > n_{cladding}$. This allows total internal reflection.

   (c) Redraw the ray diagram for light entering the cone from air at θ_{max}. Show the refraction the light undergoes as it enters the core, and then as it hits the cladding at the critical angle (the light is *just* transmitted at this point). Label angles of incidence, reflection and refraction using the standard formulae and trigonometry.

   Use your derived critical angle formula at the point where the light hits the cladding, and use Snell's law where the ray enters the core with the relevant refractive indices. Also remember standard trigonometric rules and substitutions.

6. (a) Use refractive indices to calculate the critical angle for the optical fibre, remembering that $n_{core} > n_{cladding}$ and substituting as appropriate.

   Do you need to be concerned about the multiple reflections to calculate the maximum path length, or can you make a straightforward assumption?

   The maximum path length will occur when the angle of incidence is equal to the critical angle at the cladding, at the far end of the optical fibre. Use a triangle and trigonometry to find the maximum path length.

   (b) When calculating the time difference, remember that you will need to use the speed of light *in the medium*, which you can determine using the refractive index of the medium.

   (c) Draw a square pulse. What assumption can you make in terms of the size of the gap needed to avoid interference but send information at a maximum rate? Use this to calculate the number of pulses per second (which is the maximum rate at which information is sent).

7. (a) Draw your own ray diagram. Angle a is given. Angle b should be straightforward to figure out using the law of reflection. Angles c and d are also reflections but are as yet unknown. Use standard trigonometry rules to do with angles on a straight line and angles in a triangle. You will end up with a set of simultaneous equations: see what you can eliminate and solve for c, d and e.

   (b) Consider the pattern in the angles which you have just calculated. It may help to redraw the diagram, adding more reflections.

   (c) Once again, use the pattern in the angles.

   (d) Draw a cone. Does anything change?

   (e) Calculate the wavelength of the sound waves. How does this compare to the size of the cone? What does this tell you?