

Study Focus 3 Problems - Used in Webinar on the 10th September 2025

Initial problems

You will not need a calculator for any of these problems.

1. Rectangle ABCD has an area of 120 cm^2 . Find the area of the shaded part.

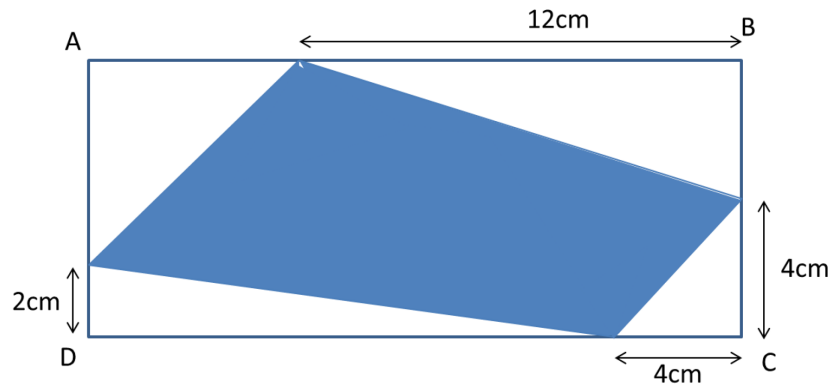


Figure 1: Shaded rectangle ABCD

2. A ball is at rest at the top of a frictionless hill. It is then given a slight nudge and speeds up to 4 ms^{-1} at the bottom of the hill. If the ball had a speed of 3 ms^{-1} at the top of the hill (instead of being stationary), what would its speed now be at the bottom?
3. A 10 by 10 by 10 cube is constructed from a thousand unit cubes. How many of the unit cubes have at least one face on the surface of the larger cube?

4. A bartender pours 100 ml of tonic water into one glass and 100 ml of gin into another. He then takes a 30 ml shot glass and scoops a shot of gin into the tonic and gives it a good mix. He then takes the shot glass and transfers 30 ml of the mix back into the gin glass. Is there more tonic in the gin glass, or more gin in the tonic glass?
5. A race has 2025 entrants, all numbered from 1 to 2025 at random. What is the probability that the first three runners to cross the finish line are numbered in ascending order?
6. An ant is at one corner of a cube of side length a . How far must it travel to reach the far corner of the cube?

Longer / harder questions

7. Each summer, when the grass grows to a certain height, the groundsman goes to the uniform cow shop and rents some uniform cows to graze on it, until it reaches a particular level. From the previous 2 years the groundsman knows that it took...
 - 6 cows, 4 days to do the job
 - 3 cows, 9 days to do the job

This year only 1 uniform cow is available. How many days will it take for the cow to do the job?

8. Sarah Connor is living off the grid and does not have access to fresh water. Following a recent impaling, she is unable to walk. Her survival is vital for the future of humanity. Each day Kyle Rees must travel from his hideout with a bucket, which he fills with water from the piranha-infested river. He must conserve energy (like every process in this universe) for the fight against the machines. What is the minimum distance he needs to travel to get to Sarah's house via the river? (See Figure 2)

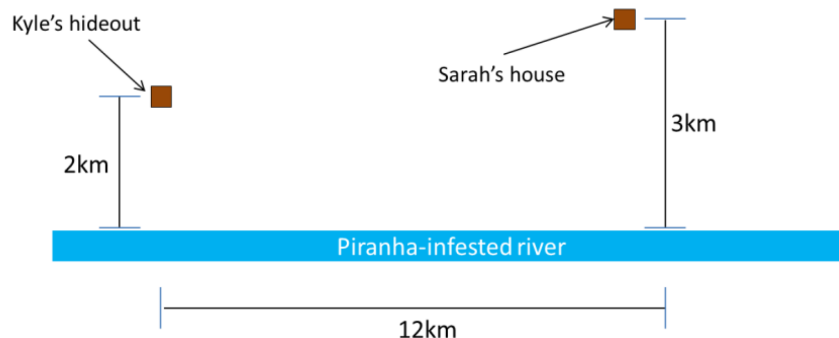


Figure 2: Sarah Connor's house and Kyle Rees' hideout

9. A person travels from Newcastle to Oxford by coach. Traffic is free-flowing and the coach's speed is only limited by whether the road is flat (63 mph), uphill (56 mph) or downhill (72 mph). The coach ride takes 4 hours from Newcastle to Oxford, but the return journey, which follows the same roads, takes an hour longer. How many miles is the coach ride between Newcastle and Oxford?

10. It's finally time for a battle to end the long running dispute between pirates and ninjas. They face off at 90 m. The pirate limps towards the ninja at 2 ms^{-1} , while the ninja glides towards the pirate at twice the speed. It is only a matter of time before they collide and crush the loyal parrot which repeatedly flies back and forth at a constant speed of 8 ms^{-1} , elastically bouncing off the two. What is the total distance the parrot travels before being crushed?

Hints and Suggestions for Study Focus 2

1. Try breaking the shaded area up into more familiar shapes. For every unshaded triangle, can you find an equivalent shaded triangle?

2. Why do you think the mass of the ball has not been specified? Have you identified which conservation law is useful here? Do not substitute numbers too early. Do you need to evaluate the height?

3. How will you deal with the edges and corners? Could you ask a different question that is almost equivalent to give the answer in one step?

4. What is the final volume of liquid in each glass? Although numbers are given here, they are not necessary to solve this.
5. Would it make a difference if there were only **2023** runners? It may be useful to not focus on the numbers: why not label the winners A,B,C?
6. Can you turn this into a 2-dimensional problem?
7. What is the reason that it doesn't take the 3 cows 8 days to do the job? Consider units: what does a cow-day represent physically? What does a cow-day per day represent physically?
8. What is the quickest way between any two points? Can you ask an equivalent question following the path of a ray of light? One approach would be to find the minimum distance using calculus, but there is a much neater solution.
9. Split the journey into 3 sections of length x , y and z . What happens to the downhill sections on the return journey? What are you trying to find in terms of x , y and z ? Do you care what each of them are? The numbers have been chosen carefully – can you tidy up awkward fractions?
10. One approach would be to find the positions where the parrot changes direction. Since we are given the speed, but asked to find distance, what else would it be useful to know?

Numerical answers can be found below.

1. 68 cm^2 ; 2. 5 ms^{-1} ; 3. 488; 4. Neither; 5. 1 in 6; 6. $\sqrt{5}a$; 7. 54 days; 8. 13 km; 9. 283.5 miles; 10. 120 m.