Thinking Like a Physicist

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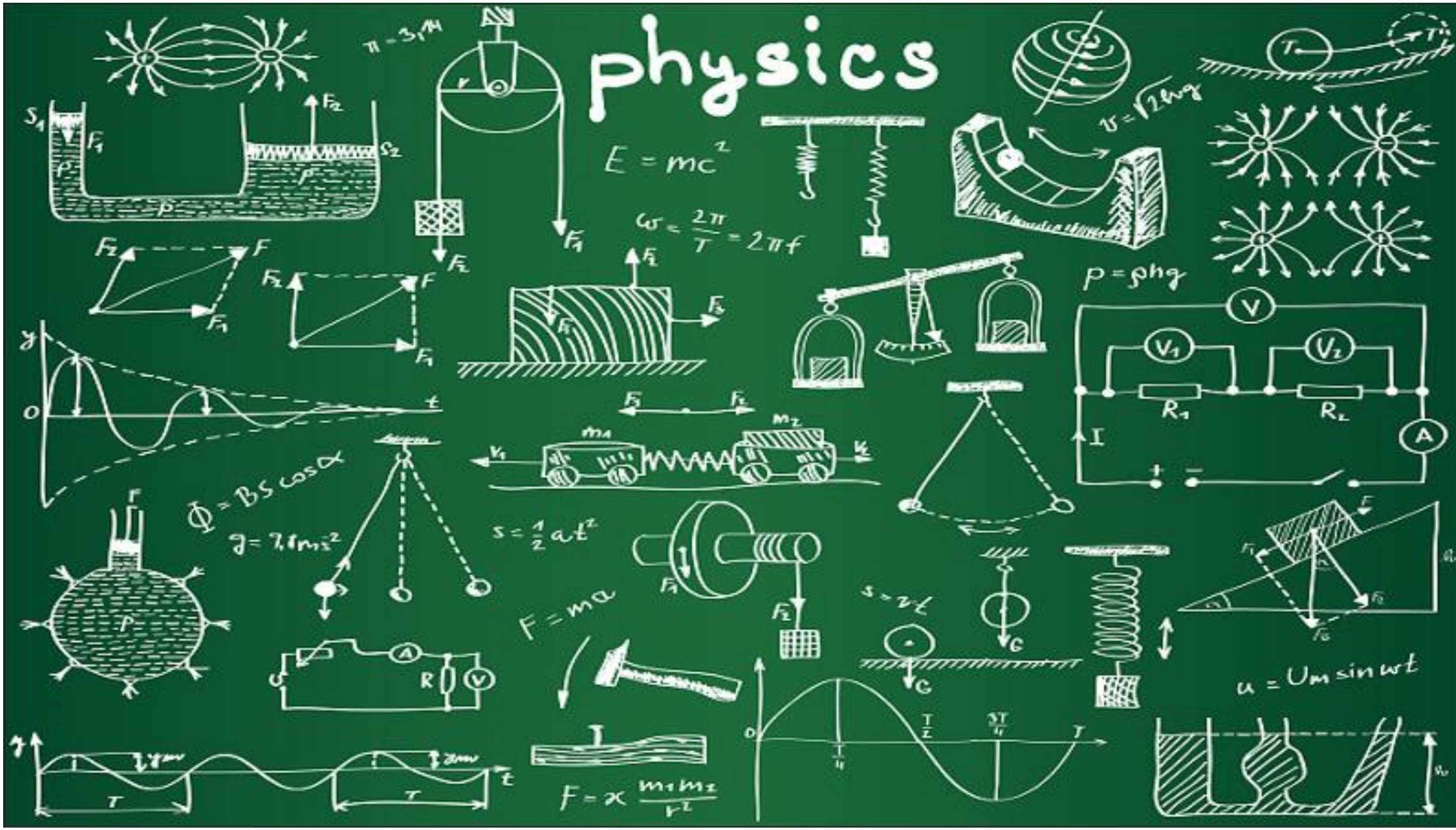

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Webinar #1: Think like a Physicist

Part I: Probabilities

1. Die rolls
2. pi versus e
3. Monty Hall
4. 3-daughters
5. Black and white cards
6. False-positive
7. Two envelopes paradox

Part II: Strategic Problem Solving Tools

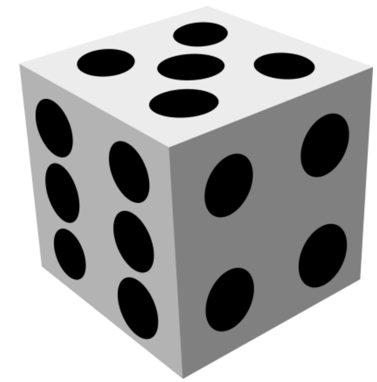
8. Rectangle
9. Ant
10. Squash v Water
11. Two Painters
12. Three Painters
13. Rats
14. Punting

1. Die Rolls Which is the most likely sequence of die rolls?

(a) 1 3 6 2 6 3 1 6 $\quad P = \left(\frac{1}{6}\right)^8$

(b) 1 3 6 5 2 6 3 1 4 $\quad P = \left(\frac{1}{6}\right)^9$

(c) 1 6 1 6 1 6 1 $\quad P = \left(\frac{1}{6}\right)^7$



Six times more likely than c) is rolling six-sixes in a row.

Misconception: It's very easy to see apparent randomness as being more likely than order. Of course, there are many more ways to be disordered than ordered, so this is much more likely and is the principle behind entropy and the eventual heat death of the universe.

A common strategy when flipping a fair coin is to keep picking the same outcome – heads or tails, in the belief that it cannot keep being the same outcome for many times in a row. What's the problem with this logic? What might be a better conclusion to draw?

2. π versus e: What is the probability of finding the consecutive sequence of digits ...123456789.... in:

a) $\pi \approx 3.14159265358979323846264338327950288419716939937510582097494\ldots$

b) $e \approx 2.71828182845904523536028747135266249775724709369995957496696\ldots$

Strange things happen when things become infinite, monkeys WILL write the complete works of Shakespeare, and any string of finite numbers will appear in irrational numbers – eventually!

In pi: The sequence starts at the **523,551,502nd** decimal digit. For context, shorter sequences like **123456** appear much earlier (at position 185,153), but as you add each digit, the rarity increases exponentially.

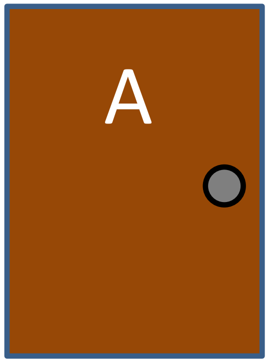
In e: Current searches of the first 1 billion digits of *e* have **not** yet found the sequence 123456789. In contrast, the sequence **0123456789** (including the zero) does not appear in the first 200 million digits of pi either, but eventually, both constants are statistically certain to contain it.

Bonus questions: solve these summations, where n is an integer

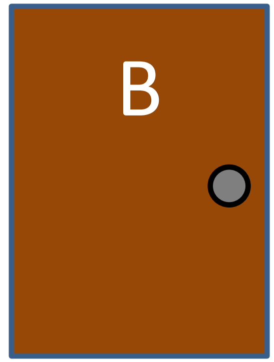
$$\sum_{n=1}^{100} n = ?$$

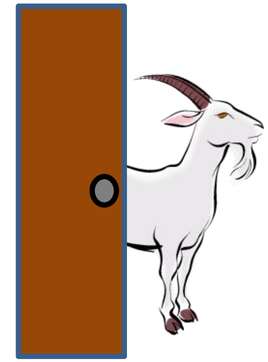
$$\sum_{n=1}^{\infty} n = ?$$

3. Monty Hall: In the prize round of a gameshow, you are presented with the choice of selecting one of three identical doors. Behind one of the doors is a car, behind the other two are goats – you really don't want a goat.



You select door B at random.





The gameshow host (who knows the location of the car), says "it's a good job you didn't pick door C" as they open it to reveal a goat.
As happens every week, you're left with the decision to stick with B or switch to A.

Misconception: Many argue that because there are now only 2 choices left, it must be a 50-50 chance, and decide to stick as the game-show host is trying to trick them.

To see through this you must realise that whatever you pick initially will not stop the game-show host showing you a goat by opening one of the other doors, so this cannot change the odds that your initial selection was correct, which was only 1 in 3.

Still not convinced? Imagine the game had 100 doors with 1 car and 99 goats. After making your selection, the host shows you 98 goats – do you really think it's a 50-50 chance between your initial random pick and door 47, which is mysteriously the only other door still closed.

4. Three daughters?: Imagine I have 3 children, and I pull out a photo from my wallet to show you a picture of one of my children at random, who just so happens to be a girl.

a) Given this, and only this information, what is the probability that I have 3 daughters?

b) Would the probability change if I were to tell you that she was my eldest, youngest or favourite child?

c) What if she was holding a trophy in the picture, and we could infer that of my 3 children, she was the most sporty?

Misconception: The implicit assumptions are that the sex of a child is independent and boy v girl is equally likely. Accepting those as fact, the usual wrong answer is 1 in 4.

The subtlety here is that I haven't told you any information about *which* of my children this is, so you **cannot** ask the question:

"what is the likelihood of having 2 daughters given the first/second/third is a girl".

The real question is:

"what is the likelihood of having 3 daughters given I do not have 3 sons?"

~~BBB~~ BBG BGB BGG GGG GGB GBG GBB

4. Three daughters?: Imagine I have 3 children, and I pull out a photo from my wallet to show you a picture of one of my children at random, who just so happens to be a girl.

b) Would the probability change if I were to tell you that she was my eldest, youngest or favourite child?

c) What if she was holding a trophy in the picture, and we could infer that of my 3 children, she was the most sporty?

Clearly, if I told you this was a photo of my first-born child, the question **does** become:
"what is the likelihood of having 2 daughters given the first/second/third is a girl".
Therefore the answer is now simply 1 in 4, as expected.

Given the 8 equally likely outcomes at the start, if we are able to identify which child this is, we gain the most information – but why must we rank them by age? Is it okay to rank them based on favouritism? This is not always fixed and can change daily, but is it valid at the time?

If I had photographs of all my children in my wallet – could we order the children by the random order in which I would pull out the photos?

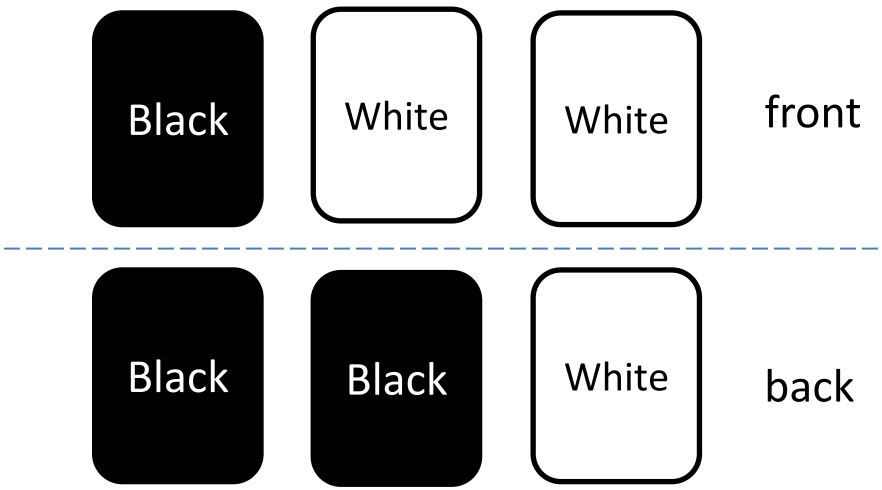
How would knowing whether or not I had 2 more photos in my wallet change the odds?

I don't think you could be 100% with your inference, say she was wearing a badge saying "my name is Anna – I'm happy to help", so you think you can rank them in alphabetical order, until you see my son Aaron… maybe thought-experiments like this is why Einstein didn't believe QM.

5. Black and White: Three cards are shuffled and laid out under a cloth. You know that one card is white on both sides, one is black on both sides and the other is black on one side and white on the other.

A card is randomly pulled from under the cloth – it's white.

Given this, what's the probability that its other side is also white?



Misconception: In the front set-up, of the two white cards, only one of them is white on the back, so it appears to be 50-50

However, you don't know whether the cards were laid out in the front or back set-up, each should be equally likely.

Laying out both options as above, we can see that if we picked a white card at random, then two-thirds of the time it would be the all-white card. Hence the answer is 2 in 3.

However, given you picked a white card, you could argue that it is twice as likely that the cards were laid out in the front set-up than the back set-up…. so the odds become 3 in 5?

6. False-positive:

1 in 100,000 people have a certain condition. The test for this is 99.9% accurate.

Bobbie administered the test and reports that it has comes back positive.

What are the odds that you actually have the condition?

Misconception: risk factors are all over medical news, but most people don't understand the actual dangers. No wonder 38% of statistics are just made up on the spot to sound convincing.

0.1% inaccurate means 1 in 1000 are diagnosed incorrectly

That means in 100,000 people the test will say that 101 people are positive

Given that only one of those 101 people actually has the condition…

you would have **less than a 1% chance** of having the condition, (from a 99.9% accurate test)!

7. Two envelopes: Envelopes, labelled A and B, contain cheques for a certain amount of money. You know for certain that one is double the value of the other

a) Let's say you pick A, you open it up and the cheque is for £250. You're now given the option to stick or twist – what should you do?

b) Now play another round with envelopes C and D. This time you pick envelope D. The cheque says it's for an amount x, will you stick or twist now?

This seems pretty straightforward, if you know one envelope contains £250, then the other must contain either £500 or £125 right?

Your initial pick was random, so you've got a 50% chance of having the higher amount – therefore the expectation value for switching is $\frac{1}{2}(£500 + £125) = £312.50$ so you should switch.

Since you can do algebra, we can just replace these example figures with x and apply the same logic – the expectation value of switching will be $\frac{1}{2}\left(2x + \frac{x}{2}\right) = \frac{5}{4}x > x$

7. Two envelopes: Envelopes, labelled A and B, contain cheques for a certain amount of money. You know for certain that one is double the value of the other

c) Final time, Envelopes E and F – take your pick. But before you even open it, aren't you just going to choose the other one anyway? Why not just pick that one at the start then?

Well, now you put it like that, it makes sense that there is no benefit in switching, so was there a flaw in the expectation value argument? The logic seems perfect.

<u>Misconception</u>: When you worked out the expectation value, the x in $2x$ and $\frac{x}{2}$ is not actually the same x – it depends which envelope you picked at the start.

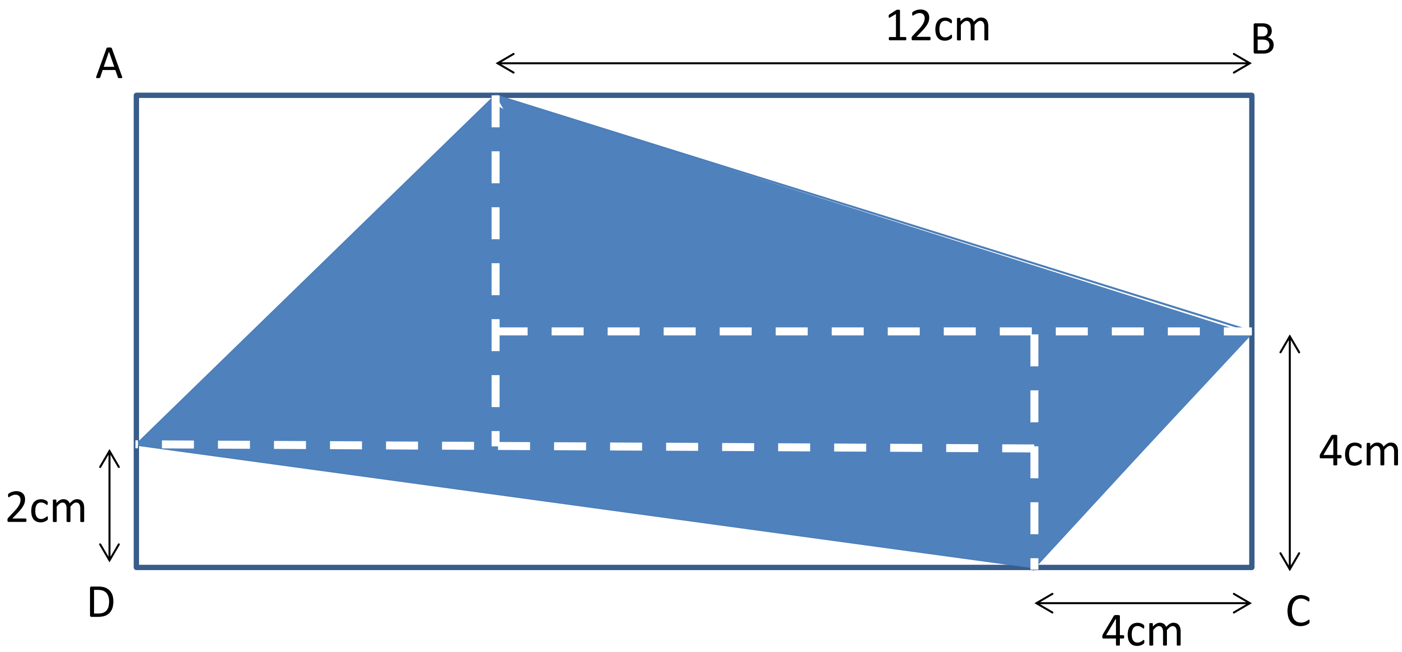
Let's label the amount in each envelope y and $2y$. Now the expectation value if you just picked at random and had to keep it would be $\frac{3}{2}y$.

The correct logic should go like this… if I picked the envelope with y and switched, I would get $2y$, but if I picked the envelope with $2y$ and switched I would get y. Each of these outcomes is equally likely.

Therefore the expectation value of switching is actually $\frac{1}{2}(2y + y) = \frac{3}{2}y$ – the same as it is without switching. But if you go back to a) with £250 – are you really going to stick?

8. Rectangle: Rectangle ABCD has an area of 120cm^2. Find the area of the shaded part.

Drawing in some dashed lines is very helpful here to split the blue area into 4 triangles and a central rectangle…



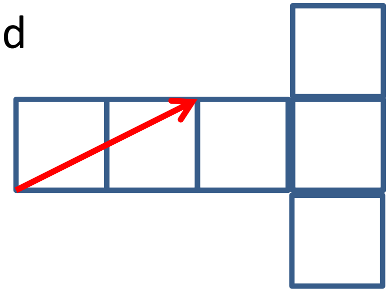
Central rectangle is 2 x 8 so has an area of 16 cm^2.

This leaves 4 triangles, which now split the remaining (120-16=104) area in half

So the area of the blue region is 104/2 + 16 = 68 cm^2

9. Ant: An ant is at one corner of a cube of side length a. How far must it travel on the surface to reach the far corner of the cube?

The shortest path across a 2D surface is a straight line, so unfold the cube into a net.



Pythagoras gives us the answer of $\sqrt{5}a$

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10. Squash v Water: You pour 100 ml of water into one glass and 100 ml of squash into another. You then take a 30 ml shot glass and scoop 30 ml of squash into the water and give it a good mix. You then take the shot glass and transfer 30 ml of the mix back into the squash glass.
Is there more water in the squash glass, or more squash in the water glass?

If you do the maths you'll discover it's the same, but there's no need to calculate this…

Each glass starts and ends with the same total volume of liquid, so any squash that is missing from one glass (and in the other) must have been replaced by water from the other and vice versa.

You may find it easier to visualise by replacing each ml with a blue or red ball.

11. Two Painters

Aled and Morgan offer to paint the outside of my house.

Aled says he'll complete the job in 3 days, whereas Morgan claims they can do the job in only 2 days.

I'm in a rush so I hire them both to work together, one going clockwise and the other anticlockwise around the house.

How long should it take to complete the job together?

It's useful to set some "common sense" boundaries to problems like this. Ask yourself…

"How long would it take if it were 2 Aleds or 2 Morgans?"

This tells us the answer we're looking for must be between 1 – 1.5 days.

To solve it properly, we must realise this is a **rate** (of work) question, so it would be useful to assign a daily rate to each painter…

Aled paints at a third of a house per day and Morgan paints at a half of a house per day.

So their combined rate is $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ of a house per day…. which leads to 1.2 days.

12: Three Painters

Kam and Chris spend 4 hours painting a fence together, then Femi helps out and the job is finished in 2 more hours.

Had Femi not helped it would have taken Kam and Chris 5 more hours to finish working together.

How long would it take Femi to have painted the fence on her own?

Although slightly more complicated, we already know the route to solve this – simply work out the rates at which each person paints the fence!

We know Kam and Chris have a combined rate of $1/9^{th}$ of the fence per hour.

So in 6 hours we can work out that they must have painted two-thirds of the fence…

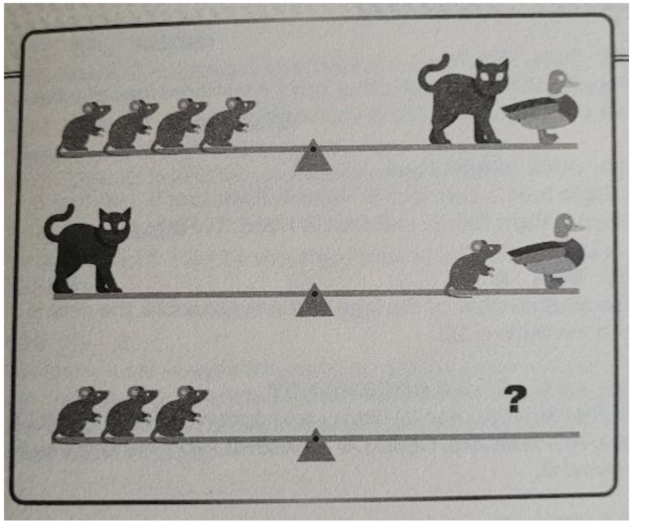
This leaves Femi a third of the fence left to paint, which she does in only 2 hours. Therefore…

She could have done the job single-handedly in only 6 hours.

13. Rats

1% Question – apparently only 1% of the general population could solve this within 30 seconds.

How many ducks would be needed to balance 3 rats?



Of course you can solve this – it's just finding the simultaneous equations and eliminating the cats to find the value of x

$$4r = c + d, \quad \text{and} \quad c = r + d \quad \text{leading to ...} \quad 3r = xd$$

... but without pen and paper and only 30 seconds to solve it – can you spot a quicker way?

Rather than eliminating the cat by substitution, if you add the first two together you'd get...

$$4r + c = c + d + r + d$$

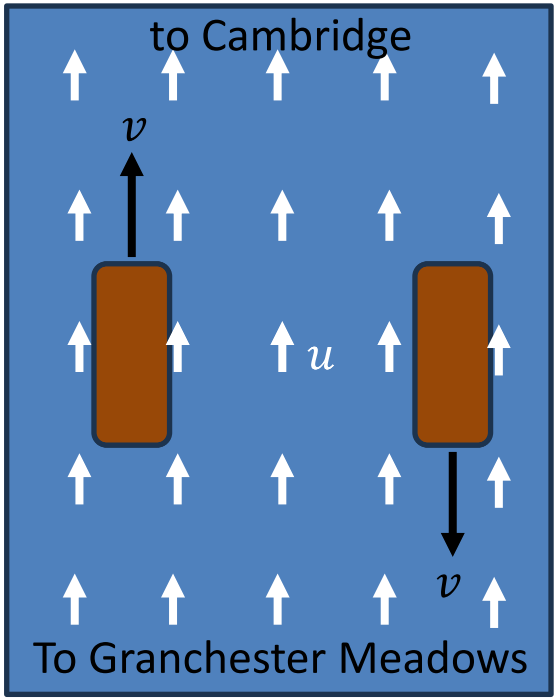
...and now all you need to do is remove a cat and rat from each side – voila!

14. Punting

Sam punts 2 miles down the river Cam from Silver Street to Granchester Meadows in 5 hours.

After re-energising with some tea and scones, they punt at the same rate during the return journey, but this only takes 3 hours.

How fast is the current?



Let's start with a diagram, with the river flowing with a constant velocity u and Sam punting at a speed v throughout.

Finding the relative velocities we see that:

$$v + u = \frac{2}{3}\,\text{mph} \qquad v - u = \frac{2}{5}\,\text{mph}$$

Subtracting these will eliminate v:

$$v + u - (v - u) = \left(\frac{2}{3} - \frac{2}{5}\right)\text{mph} \qquad \rightarrow 2u = \frac{10-6}{15} \qquad \therefore u = \frac{2}{15}\,\text{mph}$$