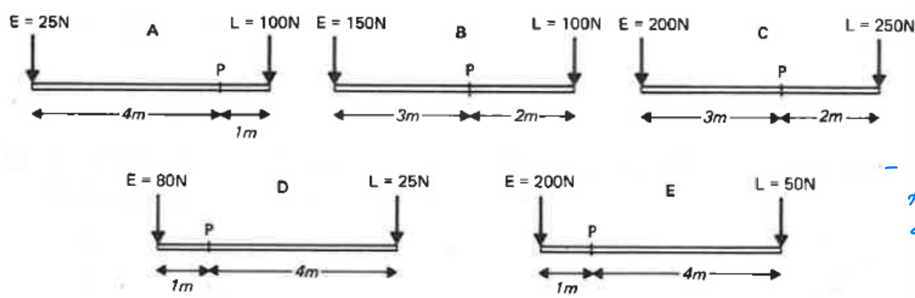
Section A

Part 1

The diagrams show five levers. P marks the pivot, E the effort and L the load. Assume all are initially at rest.



- Take clockwise to be the positive direction

1. Which lever will rotate clockwise?

- Calculate the torque on each lever

A.) $100 \times 1 - 25 \times 4 = 0$ Nm

B.) $100 \times 2 - 150 \times 3 = 200 - 450 = -250$ Nm

C.) $250 \times 2 - 200 \times 3 = -100$ Nm

D.) $25 \times 4 - 80 \times 1 = 20$ Nm

E.) $50 \times 4 - 200 \times 1 = 0$ Nm

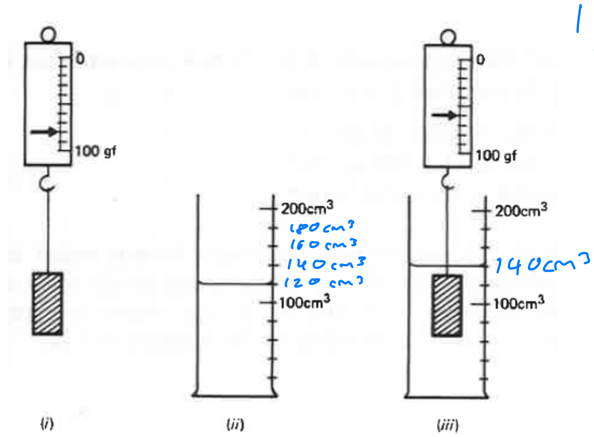
– So lever D rotates anticlockwise

2. Which lever will start to rotate most rapidly anti-clockwise?

B

as it has the largest anticlockwise moment.

The diagrams show (i) a spring balance calibrated from 0–100 gf in steps of 10 gf, weighing an object in air (ii) a measuring cylinder measuring 0–200 cm^3 in steps of 20 cm^3 and containing some water (iii) the object lowered into the water.



1gf = force that 1g feels in gravity

3. Which of these statements is/are correct?

   (a) The weight of the object is 80 gf
   (b) The volume of the water is 120 cm^3
   (c) The upthrust on the object when it is in the water is 20 gf

a.) Correct, as can be seen by reading the scale in i.)

b.) Correct, as can be seen by reading the scale in ii.)

c.) New reading in iii.) is 60gf. As weight of the object is 80g, the upthrust must be 20gf and so the statement is correct.

4. Which of these statements is/are correct?

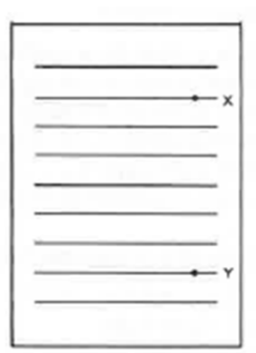
   (a) The volume of the object is 20 cm^3
   (b) The mass of the object is 80 g
   (c) The density of the object is 0.25 g cm^{-3}

a.) Increase in the cylinder reading between ii.) and iii.) is 20cm³, so a.) is correct

b.) Correct, by definition of 1gf

c.) $\rho = \frac{m}{V} = \frac{80}{20} = 4\,g\,cm^{-3}$, so c.) is incorrect

A straight wooden bar oscillates up and down in a ripple tank at 2.0 Hz and produces plane waves as shown in the diagram. Points X and Y are 60 cm apart.



5. The wavelength is
   (a) 10 cm
   (b) 30 cm
   (c) 35 cm
   (d) 60 cm
   (e) 70 cm

– 6 waves between X and Y, so

$\lambda = \frac{60cm}{6} = 10cm$

6. The velocity of the wave is
   (a) $5\,\text{cm}\,\text{s}^{-1}$
   (b) $10\,\text{cm}\,\text{s}^{-1}$
   (c) $20\,\text{cm}\,\text{s}^{-1}$
   (d) $30\,\text{cm}\,\text{s}^{-1}$
   (e) $120\,\text{cm}\,\text{s}^{-1}$

$v = \lambda f$
$= 10 \times 2$
$= 20 cms^{-1}$

Part 2

7. An estimate of the energy stored in a 1.5 V torch battery is

   (a) 10^4 J
   (b) 3×10^5 J
   (c) 10^7 J
   (d) 3×10^8 J

   Hint: consider how long you might expect such a battery to power something.

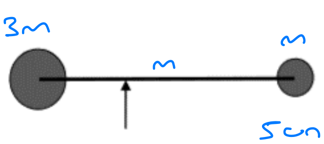
- Would expect battery to power a bulb
- Typical bulb has $P = 5W$ and would run for 10 hours

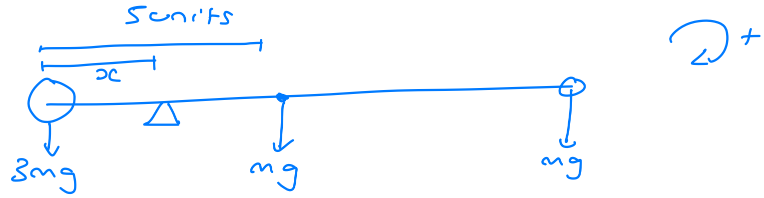
$$E = Wt$$
$$= 5 \times 10 \times 60 \times 60$$
$$= 1.8 \times 10^5$$

- So would expect answer b.)

8. Two spheres of the same density but different masses are supported with their centres at the ends of a uniform bar of length $L = 10$ units. The larger sphere has three times the mass of the smaller sphere, and the bar itself has a mass equal to the mass of the smaller sphere.

   How many units from the left hand end should the pivot be placed to balance the bar?



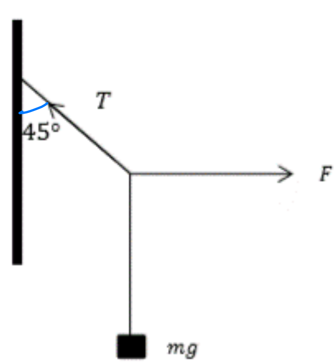


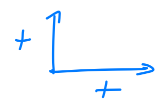
   (a) 1 unit
   (b) 2 units
   (c) 3 units
   (d) 4 units

- To balance the bar, total torque must be 0. Taking moments about the pivot:

$$-3mgx + (5-x)mg + (10-x)mg = 0$$
$$\Rightarrow \quad -3x + 5 - x + 10 - x = 0$$
$$\Rightarrow \quad 5x = 15$$
$$\Rightarrow \quad x = 3$$

An object of mass m is suspended from a light string attached to a wall, as shown in the diagram. The force F is horizontal.





A weight hanging from a string attached to a wall.

9. If the mass is doubled so that mg is replaced by $2mg$, how do forces F and T change?

   (a) F and T remain the same

   (b) T halves

   (c) F and T double

   (d) F halves

- Resolving vertically:

$$T\cos(45) = mg \quad (1)$$

- Resolving Horizontally:

$$T\sin(45) = F \quad (2)$$

(2) ÷ (1):

$$\frac{F}{mg} = \tan(45) = 1$$

$$\Rightarrow \quad F = mg$$

Sub into (1):

$$\Rightarrow \quad T = \frac{mg}{\cos(45)}$$

- So if $mg \rightarrow 2mg$

$T \rightarrow 2T$, $F \rightarrow 2T$

10. Which of these is a correct relationship?

    (a) $T \tan 45° = F$
    (b) $F \sin 45° = T$
    (c) $T \sin 45° = F$
    (d) $mg \sin 45° = T$

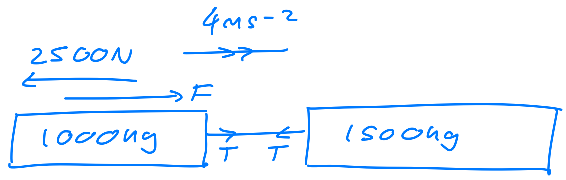
11. How are the magnitudes of F, T and mg related?

    (a) $F = mg$
    (b) $F < mg$
    (c) $F > mg$
    (d) $F + mg = T$

Part 3

12. A car of mass 1500 kg is towing a trailer of mass 1000 kg at a steady speed. The driver decides to overtake another car and accelerates at 4 m s^{-2}. If the frictional force on the trailer is 2500 N, what is the force on the towbar during the manoeuvre?

    (a) 6500 N
    (b) 8500 N
    (c) 10 000 N
    (d) 12 500 N



– Acceleration force on the trailer:

$$F = ma = 1000 \times 4 = 4000\,N$$

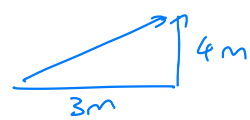
– This force is the result of the tension in the towbar and the frictional force, so:

$$-2500 + T = F$$

$$\Rightarrow \quad T = F + 2500 = 4000 + 2500 = 6500\,N$$

13. A physics lecture theatre is situated 3 m east and 4 m above reception. Calculate the minimum energy that a 60 kg receptionist would have to expend to reach the lecture theatre.

    (a) 1800 J
    (b) 2400 J
    (c) 3000 J
    (d) 4200 J



– Minimum energy = Just gravitational potential energy

$$= mgh = 60 \times 10 \times 4 = 2400\,J$$

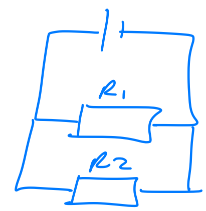
14. Two resistors R_1 and R_2 are in parallel with a potential difference V across them. The total power dissipated in the circuit is

(a)
$$V^2 \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(b)
$$\frac{V^2}{R_1 + R_2}$$

(c)
$$\frac{V^2}{(1/R_1) + (1/R_2)}$$

(d)
$$V^2(R_1 + R_2)$$



- Total resistance: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$
- Power dissipated: $P = IV = I^2V = \frac{V^2}{R_T}$

$$\Rightarrow P = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

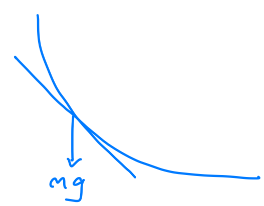
15. A drop slide in a fairground has a very steep initial slope which gradually curves into a gentler slope. If a child drops down the slide, what happens to her speed v and the magnitude of her acceleration a?

(a) v and a both increase

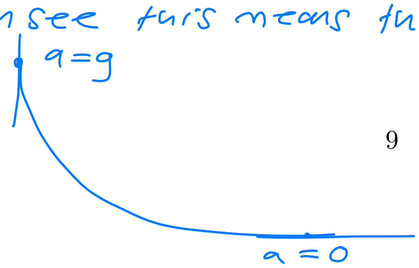
(b) v increases and a stays the same

(c) v increases and a decreases

(d) v decreases and a increases



- As the chid drops down, they are accelerated by gravity.
- GPE turns into kinetic energy, and so, v increases
- The child's acceleration at a point on the slide is the component of g parallel to the curve at that point.
- From diagram can see this means that a will decrease



16. A light-dependent resistor is connected across an ideal 12 V source and placed in the open in the middle of a desert. When is the power dissipated in the resistor highest?

(a) dawn

(b) mid-morning

(c) noon

(d) midnight

- For an LDR, more light = smaller resistance
- Power dissipated is

$$P = \frac{V^2}{R}$$

- So most power is dissipated when R is largest.
- This occurs when there is the most light, i.e at noon

17. The visible universe contains about 400 billion galaxies (where 1 billion equals 10^9). Our galaxy contains about 250 billion stars. The mass of our sun is about 2×10^{30} kg. NASA estimates that dark matter out-masses stars by about 20:1. Use this data to estimate the total mass of the visible universe.

(a) 4.2×10^{36} kg

(b) 9.5×10^{51} kg

(c) 2.0×10^{53} kg

(d) 4.2×10^{54} kg

- Total mass of normal matter:

$$M = (400 \times 10^9) \times (250 \times 10^9) \times (2 \times 10^{30})$$
$$= 2 \times 10^{53} \text{ kg}$$

- Total mass of all the matter:

$$= M + 20M$$
$$= 21M$$
$$= 21 \times 2 \times 10^{53}$$
$$= 4.2 \times 10^{54} \text{ kg}$$

18. A block of Niobium, a metal with density $8570\,\mathrm{kg\,m^{-3}}$, has sides of length 3 cm, 4 cm and 5 cm. What is the maximum pressure that can be exerted by the block when it is stood upright on one of its faces?

(a) 4.3 kPa
(b) 430 kPa
(c) 2.6 kPa
(d) 510 kPa

$P = \frac{F}{A}$

– Force the block it can exert = its weight

$\Rightarrow$ $F = mg$, $m = \rho V$

$= \rho V g$

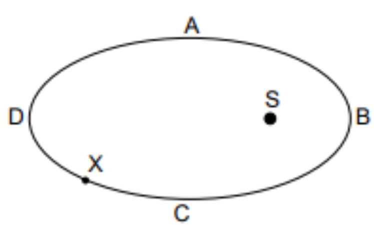
$= 8570 \times (3\times10^{-2}) \times (4\times10^{-2}) \times (5\times10^{-2}) \times 10$

$= 5.142\,N$

– Largest P $\Rightarrow$ Smallest A $= 3\,cm \times 4\,cm = 1.2\times10^{-3}\,m^2$

$\Rightarrow$ $P = \frac{5.142}{1.2\times10^{-3}} = 4285\,Pa = 4.3\,kPa$

19. The diagram shows the approximate orbit of the dwarf planet Eris (X) around the sun (S).



– Total energy, E, of the planet is constant

$E = E_k + E_p$

↳ kinetic energy + potential energy

Which of the following statements is false?

(a) Eris moves fastest at point D.
(b) Eris moves at the same speed at points A and C.
(c) Eris moves in an ellipse with the sun at one focus.
(d) The potential energy of Eris changes during the orbit.

a.) At D, E_p is the largest as the X is furthest from S. So E_k is the smallest and X will move slowest at D. Hence, a.) is incorrect.

b.) Correct, by symmetry.

c.) Correct, by Kepler's 1st law

d.) Correct, as E_p is not constant during the orbit. Only E is.

20. In quantum mechanics, the de Broglie wavelength of an object depends on its momentum according to $\lambda = h/p$ where h is Planck's constant.

Protons of charge e and mass m are accelerated from rest through a potential V. What is their de Broglie wavelength?

(a)
$$\frac{2h}{\sqrt{meV}}$$

(b)
$$\frac{h}{\sqrt{2meV}}$$

(c)
$$\frac{h}{\sqrt{meV}}$$

(d)
$$\frac{h}{eV}$$

- Velocity of protons:

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

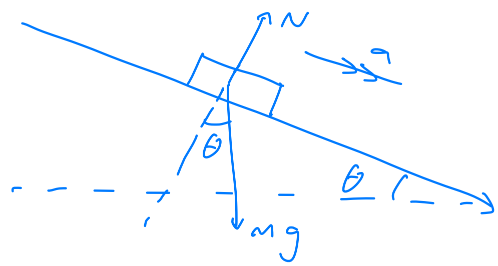
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{h}{m}\sqrt{\frac{m}{2eV}}$$

$$= \frac{h}{\sqrt{2meV}}$$

Section B

21. A truck (with a faulty handbrake) starts from rest and rolls down a slope of constant gradient. After the truck has rolled 50 m along the slope, the speedometer reads $36\,\mathrm{km\,h^{-1}}$. Calculate the gradient of the slope. (You may neglect friction and other losses.) [4]



- $36\,\mathrm{km\,h^{-1}} = 10\,\mathrm{ms^{-1}}$
- Resolving horizontally:

$$a = g\sin\theta$$

- Apply the SUVAT equations to motion along the slope:

S	U	V	A	T
50	0	10	$g\sin\theta$	

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad a = \frac{v^2}{2s}$$

$$\Rightarrow \quad g\sin\theta = \frac{10^2}{2\times 50}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{g}$$

$$= 0.1$$

$$\Rightarrow \quad \theta = 5.73^\circ$$

- Gradient of the line is:

$$m = \frac{\Delta y}{\Delta x}$$

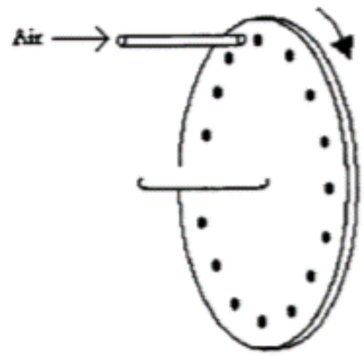
$$= \frac{50\sin\theta}{50\cos\theta}$$

$$= \tan\theta$$

$$= 0.1005\ldots$$

$$= 0.10$$

22. The diagram shows the basic idea behind a disk siren. It consists of a disk in which there are 16 equally spaced holes, all at the same distance from its axle. When a jet of air is directed at the holes and the disk is rotated at a particular constant rate, the frequency of the sound produced is 320 Hz.



(a) Calculate the frequency of the note produced when a disk containing 24 holes is rotated at 4/3 times the rate. [1]

- Using common sense, increasing the number of holes (N) increasing the rotation rate (ω) will result in a higher frequency of the produced sound.

$$\Rightarrow \quad f \propto N\omega$$

$$\Rightarrow \quad f_{new} = \frac{4}{3} \times \frac{24}{16} f_0$$

$$= \frac{4}{3} \times \frac{24}{16} \times 320$$

$$= 640 \text{ Hz}$$

(b) Explain how a sound is produced by the siren. [2]

- As the air passes through the holes, pulses of air are created.
- Sound is a series of compressions and rarefractions.
- Hence, the frequency of pulses (which depends on the number of holes and the rate of rotation) is equal to the frequency of sound produced.

23. A fibre optic cable is used to transmit signals. When a short pulse of light passes along a fibre, it spreads out. This limits the rate of transmission of the signals down the fibre.

(a) Suggest two reasons why the pulse of light might spread out. [2]

– Different wavelengths of light travel at different speeds through a fibre (as they have different refractive indices).

– Light rays can take different paths through the cable.

Hence, different light rays arrive at the end at different times => dispersion

A fibre of length 10.0 km is illuminated with red light from an LED which is turned on and off repeatedly for equal amounts of time. The speed of the pulse of light ranges from 1.95×10^8 m s^{-1} to 2.05×10^8 m s^{-1}.

(b) Calculate the range of times taken for the pulse to travel down the optical fibre. [1]

$$t_{max} = \frac{L}{V_{min}} = \frac{10 \times 10^3}{1.95 \times 10^8} = 5.13 \times 10^{-5} s$$

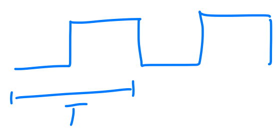
$$t_{min} = \frac{L}{V_{max}} = \frac{10 \times 10^3}{2.05 \times 10^8} = 4.88 \times 10^{-5} s$$

$$\Rightarrow \Delta t = t_{max} - t_{min} = 2.50 \times 10^{-6} s$$

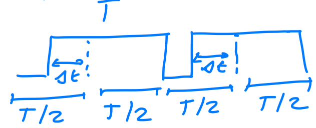
(c) What is the maximum frequency of the LED such that the pulses arrive without overlapping? [3]

– Period of pulse = $T = 1/f$

– Initial pulse looks like



– Final pulse looks like



– So can see that max freq s.t there is no overlap is

$$\Delta t = T/2$$

$$\Rightarrow f = \frac{1}{2\Delta t} = \frac{1}{2 \times 2.5 \times 10^{-6}}$$

$$= 199875$$

$$= 2.00 \times 10^5 Hz$$

(d) The wavelength which the LED emits is 1310 nm in air. Calculate the frequency of the light used. [1]

$$c = \lambda f \Rightarrow f = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^8}{1310 \times 10^{-9}}$$

$$= 2.29 \times 10^{14} Hz$$

24. While exploring Mars an astronaut finds a cave containing a liquid pool and some coloured cubes. Lacking any measuring instruments, she uses her gloved hand as a measure and determines that:

- A red cube and a green cube together are as long as her hand.
- Two green cubes and a blue cube together are twice as long as her hand.
- A red cube and a blue cube together are as long as two green cubes.
- Two red cubes, two green cubes and two blue cubes weigh the same as the astronaut in her space suit.

She puts all of the cubes in the pool and observes that all of the cubes float with half of their volume submerged. On returning to base she finds that her gloved hand is 22 cm long and she has a mass of 120 kg in her spacesuit.

Find the density of the liquid in the pool. [6]

– Let the lengths of the cubes be
r, b, g
and density of the liquid be ρ_L.

– As they all float with the same proportion of their volume density, they all have the same density, ρ_c. Hence, their masses are:

$$r^3\rho_c,\ b^3\rho_c,\ g^3\rho_c$$

– From the question, we can write:

$$r + g = 22 \quad (1)$$
$$2g + b = 44 \quad (2)$$
$$r + b = 2g \quad (3)$$
$$2\rho_c(r^3 + b^3 + g^3) = 120 \quad (4)$$
$$2\rho_c = \rho_L \quad (5)$$

(using upthrust = weight of fluid displaced, and upthrust = weight of cube, as the forces balance)

"(1) + (2)",
$$r + b + 3g = 66$$
"sub in (3)",
$$5g = 66$$
$$\Rightarrow \boxed{g = 13.2\text{cm}}$$
"sub g into (1)",
$$r + 13.2 = 22$$
$$\Rightarrow \boxed{r = 8.8\text{cm}}$$

"sub g into (2)",
$$2\times(13.2) + b = 44$$
$$\Rightarrow \boxed{b = 17.6\text{cm}}$$

rSub r, b and g into 4,

$$2\rho c(13.2^3 + 8.8^3 + 17.6^3) = 120$$

$$\Rightarrow \quad 16866.432\,\rho c = 120$$

$$\Rightarrow \quad \rho c = 7.11 \times 10^{-3}\,kg\,cm^{-3}$$

$$\Rightarrow \quad \boxed{\rho c = 7.1\,g\,cm^{-3}}$$

rSub ρc into 5,

$$\rho_L = 2\rho c$$

$$= 2 \times 7.1$$

$$\Rightarrow \quad \boxed{\rho_L = 14.2\,g\,cm^{-3}}$$

Section C

25. In this question you will use a simple model to estimate how the energy used by a car depends on its design and how it is driven. Begin by neglecting air and ground resistance and assume that the car travels at a constant velocity between regular equally spaced stops.

    (a) A stationary car of mass m is rapidly accelerated to a velocity v, driven for a distance s, and is then rapidly brought to a halt by its brakes. Calculate the energy dispersed by the brakes. [1]

- Energy dissipated = K.E of the car

$$= \frac{1}{2} m v^2$$

    (b) Assuming the car restarts immediately, calculate the time between subsequent stops and hence the average power dissipated. [2]

- Neglecting the time it takes to accelerate, time between stops is:

$$t = \frac{s}{v}$$

- Average power:

$$P_{\text{breaks}} = \frac{E}{t} = \frac{1}{2} m v^2 \times \frac{v}{s}$$

$$= \frac{m v^3}{2 s}$$

    (c) Hence, or otherwise, calculate the energy used in travelling a total distance d. [2]

- On average, car has speed v
- So time to travel d is:

$$t = \frac{d}{v}$$

- Then

$$E_{\text{breaks}} = P_{\text{breaks}}\, t$$

$$= \frac{m v^3}{2 s} \times \frac{d}{v}$$

$$= \frac{m d v^2}{2 s}$$

(d) Taking $m = 1000\,\mathrm{kg}$, $v = 10\,\mathrm{m\,s^{-1}}$ and $s = 100\,\mathrm{m}$, calculate the energy used in travelling 1 km. What would be the effect of doubling the speed to $20\,\mathrm{m\,s^{-1}}$? [3]

$$E_{brakes} = \frac{1000 \times 1\times10^3 \times 10^2}{2\times100}$$
$$= 5\times10^5\,\mathrm{J}$$
$$= 500\,\mathrm{kJ}$$

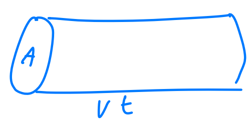
- As $E_{brakes} = \frac{mdv^2}{2s}$, if the speed is doubled $(v \to 2v)$,

$$E \to 4E$$

Now consider the effect of air resistance. This can be estimated by assuming that the car has to accelerate all the air it travels through to the same average velocity as itself. (You may ignore the rapid random motion of individual air molecules in this calculation.)

(e) Treating the car as a disc with cross sectional area A travelling at a velocity v, calculate the volume of air swept out in a time t. If the air has density D, calculate the kinetic energy transferred to the air in this time, and hence the power needed to overcome the air resistance. [5]

- In a time t, volume swept by car is



$$V = Avt$$

- Total mass of air swept is

$$M = DV = DAvt$$

- So kinetic energy transferred to air is

$$E_{air} = \frac{1}{2}Mv^2$$
$$= \frac{1}{2}DAvt\,v^2$$
$$= \frac{DAtv^3}{2}$$

- Then power required is

$$P_{air} = \frac{E_{air}}{t}$$
$$= \frac{DAv^3}{2}$$

(f) Taking $A = 1\,\mathrm{m}^2$, $v = 10\,\mathrm{m\,s^{-1}}$ and $D = 1\,\mathrm{kg\,m^{-3}}$, calculate the energy used in travelling 1 km. [2]

- Travel 1km, so $vt = 1\,\mathrm{km} = 1\times10^3\,\mathrm{m}$
- From e.) $E_{air} = \frac{DA(vt)v^2}{2} = \frac{DAdv^2}{2}$

$$= \frac{1\times1\times1\times10^3\times10^2}{2}$$

$$= 5\times10^4\,\mathrm{J} = 50\,\mathrm{kJ}$$

- From d.) $E_{breaks} = 500\,\mathrm{kJ}$
- So total energy used $= 500+50$ $= 550\,\mathrm{kJ}$

(g) Using the data above, calculate the distance between stops at which the energy dissipated in the brakes is the same as that lost to air resistance. [2]

$$E_{breaks} = E_{air}$$

$$\Rightarrow \frac{DAdv^2}{2} = \frac{mdv^2}{2s}$$

$$\Rightarrow DA = \frac{m}{s}$$

$$\Rightarrow s = \frac{m}{DA}$$

$$= \frac{1000}{1\times1} = 1000\,\mathrm{m}$$

(h) Comment on the significance of these calculations for the design of cars optimised for driving in cities and cars optimised for driving on highways. [3]

$$P_{breaks} = \frac{mv^3}{2s}, \quad P_{air} = \frac{DAv^3}{2}$$

- In a city would expect a small v and s. Hence, from the above, would expect

$$P_{breaks} \gg P_{air}$$

↳ Breaks dissipate the most energy, so to minimise energy losses, will want to minimise m

- On highways, v and s would be big and so would expect $P_{air} \gg P_{breaks}$

↳ Hence, to minimise losses, should minimise P_{air}, which can be done by minimising A.