

Section A

Part 1

1. When undoing a tight nut on a car, a mechanic may slide a long steel tube over the spanner so as to make it longer. What effect does this have?
 - (a) Increases the effort needed to undo the nut
 - (b) Reduces the moment (turning effect) of the force she applies
 - (c) Increases the mechanical advantage

A girl weighing 400 N runs up a flight of stairs of vertical height 5 m in 4 seconds.

2. Her increase in gravitational potential energy is

(a) 1600 N

(b) 1600 J

(c) 1600 W

(d) 2000 J

(e) 2000 W

$$\begin{aligned} E_p &= mgh \\ &= Wh, \quad W = \text{weight} \\ &= 400 \times 5 \\ &= 2000 \text{ J} \end{aligned}$$

3. Her rate of working against gravity is

(a) 320 W

(b) 400 W

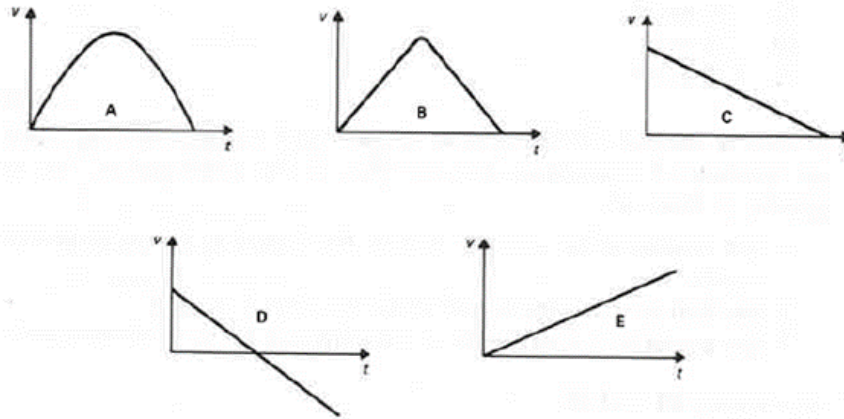
(c) 500 W

(d) 2000 W

(e) 8000 W

$$\begin{aligned} P &= \frac{E_p}{t} \\ &= \frac{2000}{4} \\ &= 500 \text{ W} \end{aligned}$$

The diagrams show several velocity-time graphs.



For each question, choose the velocity-time graph which most nearly represents the example described.

4. A car braking to a halt.

— Deceleration until $v=0$

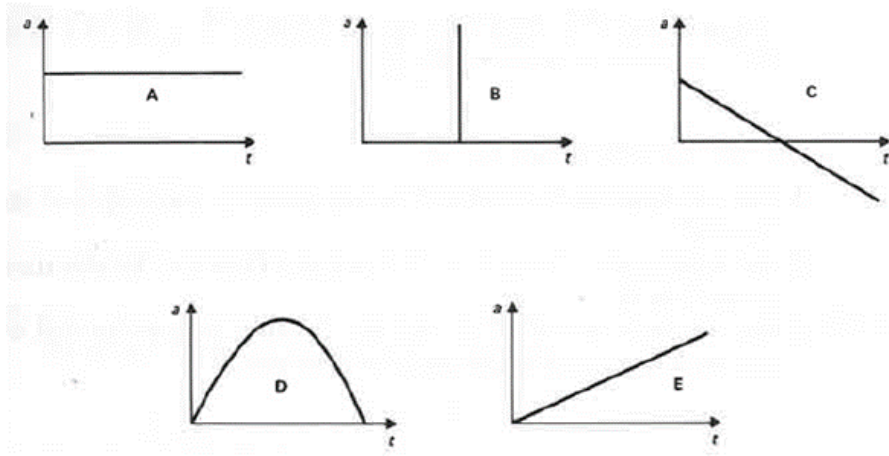
$\Rightarrow$ C

5. A ball thrown vertically upwards and then caught as it returns.

— Initial velocity > 0 , final velocity < 0
 — Acceleration is constant and equal to $-g$

$\Rightarrow$ D

6. For the ball in Question 5, the corresponding acceleration-time graph is which of the following?



- Acceleration = gradient of v - t graph

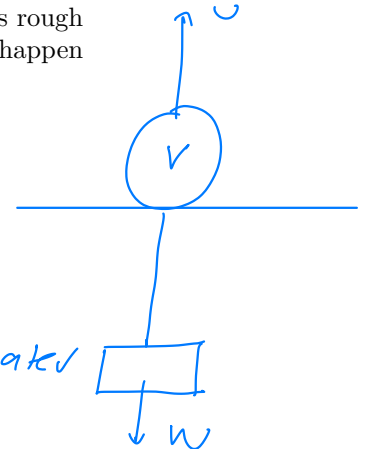
$\Rightarrow A$

$\hookrightarrow$ Assuming a = modulus of the acceleration

Part 2

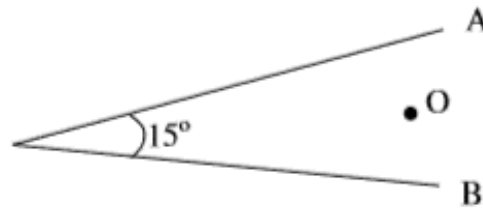
7. A heavy piece of apparatus used to measure sea salinity is attached by a short rope to a flexible rubber balloon filled with air. The balloon should sit on the surface of the sea. On one occasion, too little air is put in and the balloon sits just below the surface of the sea. The sea becomes rough and the balloon sinks down just a little further. What is likely to happen to the apparatus?

- (a) It sinks to the ocean floor
(b) It sinks to some reasonable depth and stays there
(c) It rises again and sits below the surface of the sea
(d) It remains at the level it has sunk to



- Apparatus floats due to upthrust of the balloon pulling it up. $U = \rho V$, ρ = density of water
- Pressure felt by the balloon at depth h is $p = \rho h g$
- If balloon is pushed deeper $\Rightarrow$ higher P on balloon
 $\Rightarrow$ Balloon shrinks
 $\Rightarrow$ Smaller upthrust
 $\Rightarrow U < W$
 $\Rightarrow$ Apparatus sinks

8. Two plane mirrors are at an angle of 15° as shown in the diagram. A small object O is placed between them at an equal distance from mirror A and from mirror B . How many images can be seen (including the original)? (You can fit your eyeball between the mirrors if required.)



- (a) None
(b) 24
(c) 36
(d) 48

Handwritten calculations and reasoning:

- 180° and 90° diagrams showing image formation.
- $\cdot \mid \cdot = 2$
- $\cdot \mid \cdot = 4$
- Following this trend, number of images seen is

$$\frac{360}{15} = 24$$

9. A toy boat floats in a tank of water which is carefully balanced on a block of wood. If the boat slowly drifts to the right across the tank in the diagram below, what would likely happen to the tank of water in which it is floating?



- (a) The tank will tip so that the right hand side drops down
 (b) The tank remains balanced
 (c) The tank will tip so that the left hand side drops down
 (d) It depends on how slowly the boat drifts across

- "Slow" = slow enough s.t it won't disturb the surface of the water
 - Upthrust on the boat will always balance its weight (i.e., internal forces all balance)
 - So there is no force to provide a torque
 $\Rightarrow$ tank remains balanced

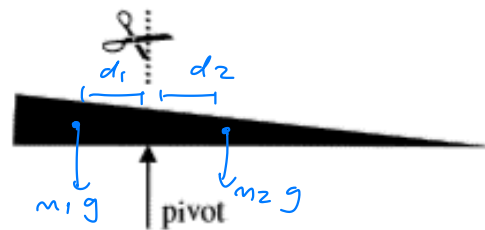
10. In the LEP accelerator at CERN, before the LHC was constructed in the same tunnels, an electron could be accelerated until it reached a total energy of about 400 GeV. Estimate which of the examples below has a similar amount of energy.

- (a) An oxygen molecule moving at 500 m s^{-1} (a typical speed at room temperature)
 (b) A snail crawling at 2 mm s^{-1}
 (c) A house brick that has fallen 100 m
 (d) An airliner cruising at 500 km h^{-1}

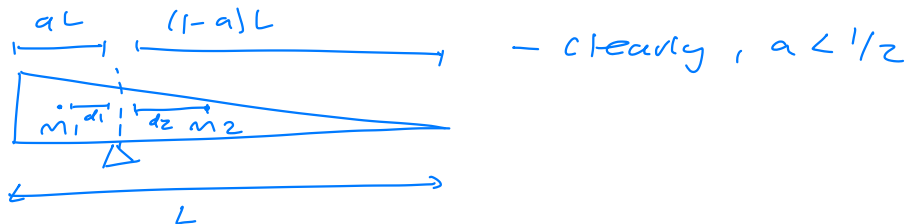
$$400 \text{ GeV} = 400 \times 10^9 \times 1.60 \times 10^{-19} \\ = 6.4 \times 10^{-8} \text{ J}$$

a.) $m = 150$ $\Rightarrow E = \frac{1}{2} m v^2 = \frac{1}{2} \times 15 \times 1.67 \times 10^{-27} \times 500^2 \sim 10^{-21} \text{ J}$
 b.) $m = 10 \text{ g}$ $\Rightarrow E = \frac{1}{2} \times 10 \times 10^{-3} \times (2 \times 10^{-3})^2 \sim 10^{-8} \text{ J}$
 $\hookrightarrow$ what we need

11. A wedge-shaped beam of uniform density wood balances on a pivot. If the wedge is cut in half vertically at the balance point, and each half is weighed on a balance, which of the following statements is true?



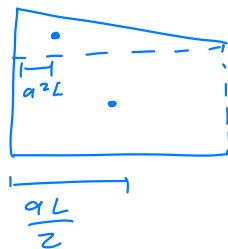
- (a) The left hand half has a greater mass
 (b) The right hand half has a greater mass
 (c) The two halves could have the same or different masses
 (d) The two halves must have the same mass



— Apply the above to find the COM of the LHS



— Now consider the RHS



— So COM of RHS is located a distance d_1 from the pivot, where

$$d_1 < \frac{aL}{2}$$

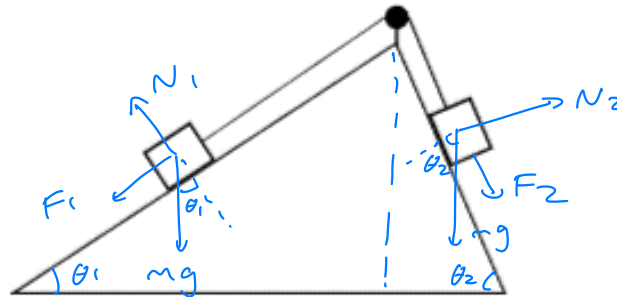
— But as $a < \frac{1}{2} \Rightarrow \frac{aL}{2} < a(1-a)L$

$$\Rightarrow d_1 < d_2$$

$$\Rightarrow m_1 > m_2$$

as for wedge to balance, require $m_1 d_1 = m_2 d_2$

12. Two identical masses are shown hanging from a light string passing over a frictionless pulley. There is no friction between the masses and the slopes, which are at different angles. The masses are released from rest.



Which of the following statements is true?

- (a) The masses remain at rest
- (b) The masses slide off to the left
- ☒ (c) The masses slide off to the right
- (d) The masses move to a balanced position

- Resultant force on the left mass is along the slope and has magnitude

$$F_1 = mg \sin \theta_1$$

- Resultant force on the right mass is also along the slope but has magnitude

$$F_2 = mg \sin \theta_2$$

- As

$$\theta_2 > \theta_1 \Rightarrow F_2 > F_1$$

$\Rightarrow$ masses slide right

13. The neutral point between the Earth and the Moon is the point where the gravitational pull of the Moon is equal to the gravitational pull of the Earth. If the energy a 1000 kg spacecraft needs in order to reach the neutral point from Earth is 6.0×10^{10} J and to reach the neutral point from the Moon is 0.25×10^{10} J, what is the minimum energy needed to send a 1 kg rock from the Moon to the Earth?

- ☒ (a) 0.25×10^7 J
- (b) 5.75×10^7 J
- (c) 6.0×10^7 J
- (d) 6.25×10^7 J



- When spacecraft travels from the moon, it only requires 0.25×10^{10} J
 c) Once it reaches the N.P. it will fall down to the Earth

- As $m_{\text{rock}} = 1 \text{ kg}$, energy required would be $\frac{0.25 \times 10^{10} \text{ J}}{1000} = 0.25 \times 10^7 \text{ J}$

Part 3

14. A bullet with a mass of 10 g is fired at a velocity of 400 ms^{-1} into a cubical tank of water 2 m on each side and is brought to a halt by friction. Given that the heat capacity of water is $4.2 \text{ kJ K}^{-1} \text{ kg}^{-1}$ and its density is 1000 kg m^{-3} , calculate the temperature rise of the water in the tank.

- (a) $2.4 \times 10^{-5} \text{ K}$
 (b) $4.8 \times 10^{-5} \text{ K}$
 (c) $1.9 \times 10^{-4} \text{ K}$
 (d) $2.4 \times 10^{-2} \text{ K}$

- Change in energy of bullet:

$$\begin{aligned}
 E &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2} \times 10 \times 10^{-3} \times 400^2 \\
 &= 800 \text{ J} \\
 &= \text{Energy gained by the water}
 \end{aligned}$$

$$\begin{aligned}
 E &= mc\Delta\theta, \quad m = \rho V \\
 \Rightarrow \Delta\theta &= \frac{E}{mc} \\
 &= \frac{E}{\rho Vc} \\
 &= \frac{800}{1000 \times 2^3 \times 4.2 \times 10^3} \\
 &= 2.4 \times 10^{-5} \text{ K}
 \end{aligned}$$

15. A pressure cooker has an escape valve that is essentially a 125 g weight resting on a circular hole of radius 1 mm. What pressure will lift the weight off the hole?

- (a) 400 Pa
 (b) 40 kPa
 (c) 400 kPa
 (d) 400 MPa

$$\begin{aligned}
 P &= \frac{F}{A} \\
 &= \frac{mg}{A} \\
 &= \frac{125 \times 10^{-3} \times 10}{\pi (1 \times 10^{-3})^2} \\
 &= 400000 \text{ Pa} \\
 &= 400 \text{ kPa}
 \end{aligned}$$

16. A 9 V battery is connected across a $100\ \Omega$ resistor. Given that the charge on an electron is 1.6×10^{-19} C, what is the number of electrons passing through the resistor every second?

(a) 5.6×10^{17}

(b) 6.9×10^{19}

(c) 5.6×10^{21}

(d) 6.9×10^{15}

$$Q = It, \quad I = \frac{V}{R}$$

$$\Rightarrow Q = \frac{Vt}{R}$$

$$\Rightarrow N = \frac{Q}{e}$$

$$= \frac{Vt}{eR}$$

$$= \frac{9 \times 1}{1.6 \times 10^{-19} \times 100}$$

$$= 5.6 \times 10^{17}$$

17. An alien civilisation is in the business of building custom solar systems. Their basic model has five planets in circular orbits at distances D that are perfect square multiples of a basic length, so that they are in the ratio $1 : 4 : 9 : 16 : 25$. For this model, the year lengths Y of the planets are in ratios $1 : 8 : 27 : 64 : 125$. How are D and Y related?

(a)

$$\frac{Y}{D} = k$$

(b)

$$D = k\sqrt{Y}$$

(c)

$$Y = kD\sqrt{D}$$

(d)

$$\frac{Y^3}{D^2} = k$$

$$Y^3 = D^2 k$$

- Consider 1st and 2nd planets

$$\frac{D_1}{D_2} = \frac{1}{4}, \quad \frac{Y_1}{Y_2} = \frac{1}{8}$$

$$\Rightarrow \left(\frac{D_1}{D_2}\right)^{1/2} = \left(\frac{Y_1}{Y_2}\right)^{1/3}$$

$$\Rightarrow \frac{Y_1}{Y_2} = \left(\frac{D_1}{D_2}\right)^{3/2}$$

$$\Rightarrow Y = k D^{3/2} = k D \sqrt{D}$$

18. When an ideal gas is heated in a container of fixed volume then:

- (a) the pressure and density both rise
- (b) the pressure rises and the density falls
- (c) the pressure rises and the density stays the same
- (d) the pressure stays the same and the density falls

- Ideal gas equation:

$$pV = Nk_B T$$

- V is constant

- Assuming container is closed N is also constant
 $\Rightarrow$ density is constant

- Now $p \propto T$, so pressure rises as T rises

19. A Martian attempts to measure her mass using a set of bathroom scales in her house on Mars and obtains a reading of 93 kg. Unfortunately, her scales were designed for use on Venus. Given that the gravitational strengths at the surfaces of Mars and Venus are 3.8 N kg^{-1} and 8.8 N kg^{-1} respectively, what is her true mass?

- (a) 40 kg
- (b) 106 kg
- (c) 215 kg
- (d) 245 kg

- $m = \text{True mass of Martian}$

$$W_M = mg_M, W_V = mg_V$$

- Venus scales, on Mars, would give the mass

$$\bar{m} = \frac{W_M}{g_V}$$

$$= \frac{mg_M}{g_V}$$

$$\Rightarrow m = \frac{\bar{m} g_V}{g_M}$$

$$= 93 \times \frac{8.8}{3.8}$$

$$= 215 \text{ kg}$$

20. When a metal bar is cooled, it contracts. Which of the following is true?

- (a) The density and mass increase
- (b) The density increases and the mass remains constant
- (c) The density and mass are unchanged
- (d) The mass remains constant and the density decreases

- Mass of the metal bar is constant
- As it contracts, its volume will decrease
- $\rho = \frac{m}{V} \Rightarrow \rho$ will increase

Section B

21. The dwarf planet Pluto (radius 1180 km) is populated by three species of purple caterpillar. Studies have established the following facts:

- A line of 5 mauve caterpillars is as long as a line of 7 violet caterpillars.
- A line of 3 lavender caterpillars and 1 mauve caterpillar is as long as a line of 8 violet caterpillars.
- A line of 5 lavender caterpillars, 5 mauve caterpillars and 2 violet caterpillars is 1 m long in total.
- A lavender caterpillar takes 10 s to crawl the length of a violet caterpillar.
- Violet and mauve caterpillars both crawl twice as fast as lavender caterpillars.

How long would it take a mauve caterpillar to crawl around the equator of Pluto? [6]

- Let the lengths of the caterpillars be M, L, V

- From the question:

$$5M = 7V \quad (1)$$

$$3L + 1M = 8V \quad (2)$$

$$5L + 5M + 2V = 1 \quad (3)$$

↳ (1) → (3),

$$\Rightarrow 5L + 9V = 1 \quad (4)$$

↳ $5 \times (2)$,

$$15L + 5M = 40V$$

↳ Sub in (1),

$$15L = 33V$$

$$\Rightarrow 5L = 11V \quad (5)$$

↳ Sub into (4),

$$\Rightarrow 20V = 1$$

$$\Rightarrow \boxed{V = 0.05\text{m}}$$

- Let the speeds of the caterpillars be U_r, U_L, U_m

- From the question:

$$U_L = \frac{V}{10} \\ = 0.005 \text{ ms}^{-1}$$

$$U_r = U_m = 2U_L = 0.01 \text{ ms}^{-1}$$

- Equator of Pluto is:

$$E = 2\pi r \\ = 2\pi \times 1180 \times 10^3 \\ = 7.41 \times 10^6 \text{ m}$$

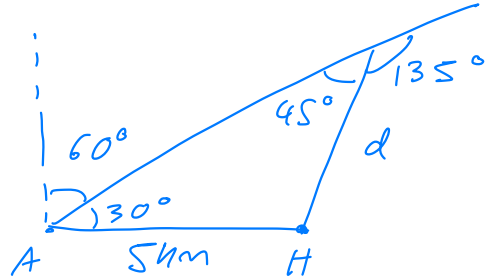


$$r = 1180 \times 10^3 \text{ m}$$

- Time for mave to walk around equator is:

$$t = \frac{E}{U_m} \\ = \frac{7.41 \times 10^6}{0.01} \\ = 7.41 \times 10^8 \text{ s}$$

22. A pilot takes off from an airfield 5 km west of her house and flies in a direction 60° east of north. After 5 minutes, she sees that the direction to her house is now at an angle 135° to her course. How far away is she from her house? [4]



$$\frac{\sin(45)}{5} = \frac{\sin(30)}{d}$$

$$\begin{aligned} \Rightarrow d &= \frac{\sin(30)}{\sin(45)} \times 5 \\ &= \frac{5\sqrt{2}}{2} \text{ km} \end{aligned}$$

23. This problem will consider the possibility of storing electrical energy in capacitors which are made up of two parallel metal plates, each of area A , and separated by a thickness d of a dielectric with electrical permittivity p . A capacitor will store a charge $q = CV$, where the capacitance C is given by

$$C = \frac{pA}{d}$$

and V is the voltage across the capacitor.

- (a) Given that the work done in charging the capacitor is

$$W = \frac{q^2}{2C}$$

show that the work done to charge a parallel plate capacitor is

$$W = \frac{pAV^2}{2d}$$

[2]

$$\begin{aligned} W &= \frac{q^2}{2C} \\ &= \frac{(CV)^2}{2C} \\ &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times \frac{pA}{d} \times V^2 \\ &= \frac{pAV^2}{2d} \end{aligned}$$

This work is equal to the energy stored in the capacitor. In practice, a capacitor is limited by the *breakdown voltage* of its dielectric. This breakdown voltage is proportional to its thickness: $V_{MAX} = Bd$, where B is a constant that depends on the material.

- (b) Determine how the maximum energy stored in a capacitor depends on the mass m of the dielectric, its density D and other relevant constants. [3]

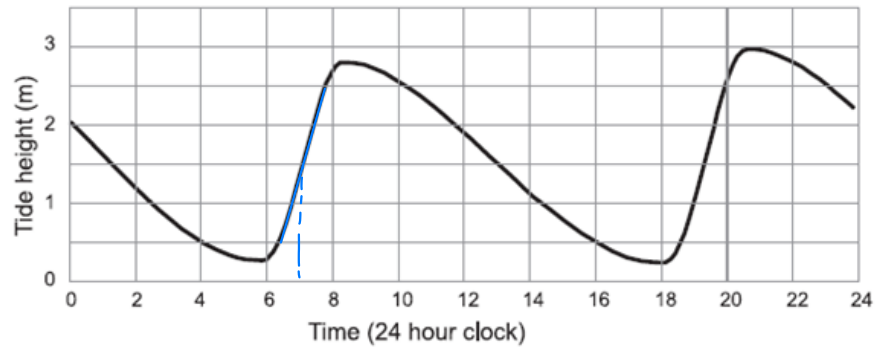
$$\begin{aligned}
 W_{max} &= \frac{pAV_{max}^2}{2d} \quad , \quad D = \frac{m}{Ad} \\
 &= \frac{pAB^2d^2}{2d} \quad \Rightarrow Ad = \frac{m}{D} \\
 &= \frac{pAdB^2}{2} \\
 &= \frac{pmB^2}{2D}
 \end{aligned}$$

- (c) For a dielectric with $p = 2 \times 10^{-11} \text{ F m}^{-1}$, $B = 2 \times 10^7 \text{ V m}^{-1}$ and $D = 1000 \text{ kg m}^{-3}$ (approximate values for plastic dielectrics), calculate the maximum energy that can be stored in a capacitor with a mass of 1 kg. Comment on the practicality of using a capacitor of this type to smooth the output from a 1 kW domestic wind turbine in a gusty area. [2]

$$\begin{aligned}
 W_{max} &= \frac{2 \times 10^{-11} \times 1 \times (2 \times 10^7)^2}{2 \times 1000} \\
 &= 4 \text{ J}
 \end{aligned}$$

- $1 \text{ kW} = 1000 \text{ J s}^{-1}$, which is much larger than 4 J
- So this capacitor would not be effective in smoothing the output of a wind turbine.

24. The graph below shows tide heights at Wadebridge on 7th July 2008.



At what time in the morning (to the nearest hour) is the tide height changing most rapidly? What is the rate of change in cm per minute at this time? [3]

- Rate of change of tide height = gradient of the graph
- Max gradient = at 7:00

$$\text{gradient} = \frac{\Delta h}{\Delta t}$$

$$= \frac{2\text{ m}}{1\text{ hr } 10\text{ min}}$$

$$= \frac{2 \times 10^{-2} \text{ cm}}{70 \text{ min}}$$

$$= 2.86 \times 10^{-4} \text{ cm min}^{-1}$$

$$= 3 \times 10^{-4} \text{ cm min}^{-1}$$

Section C

25. This question covers the mathematical treatment of an ideal pendulum, in which the pendulum bob is modelled as a point mass on the end of a rigid rod of negligible mass. In this problem you will consider the behaviour of more complex types of pendulum. You will be given all the information you need in the sections below.

For a general pendulum of any shape and size, the period P is given by

$$P = 2\pi \sqrt{\frac{I}{gML_{CM}}}$$

where g is the acceleration due to gravity, M is the total mass of the pendulum, L_{CM} is the effective length of the pendulum (defined as the distance from the pivot to the centre of mass) and I is the *moment of inertia* around the pivot point.

For a point mass m fixed at a distance r from the pivot, $I = mr^2$, while for a uniform rod of mass m and length r attached to the pivot at one end, $I = \frac{1}{3}mr^2$. For more complex objects, the total moment of inertia can be calculated by adding together values for the component parts.

- (a) Identify L_{CM} and I for an ideal pendulum of length L with a bob of mass M , and hence calculate its period. [3]

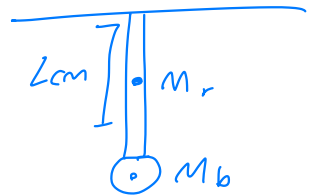
$$\begin{aligned} L_{CM} &= L \\ I &= ML^2 \\ \Rightarrow P &= 2\pi \sqrt{\frac{ML^2}{gML}} \\ &= 2\pi \sqrt{\frac{L}{g}} \end{aligned}$$

- (b) Repeat the calculation in part (a) for a pendulum made from a uniform rod of mass M and length L . [3]

$$L_{cm} = L/2, \quad I = \frac{1}{3} M L^2$$

$$\Rightarrow P = 2\pi \sqrt{\frac{\frac{1}{3} M L^2}{g M L/2}} \\ = 2\pi \sqrt{\frac{2L}{3g}}$$

- (c) Now consider the case of a real pendulum, with a bob of mass M_b (which you may treat as a point mass) attached to the pivot using a uniform rod of mass M_r and length L , and find the period in this case. Show that the result for a real pendulum reduces to the results for an ideal pendulum and a rod pendulum by taking appropriate limits. [7]



$$I_{cm} = M_b L^2 + \frac{1}{3} M_r L^2 \\ = (M_b + \frac{1}{3} M_r) L^2$$

$$L_{cm} = \frac{\sum m_i r_i}{\sum m_i} \\ = \frac{\frac{L}{2} M_r + L M_b}{M_r + M_b}$$

$$= \frac{L (M_r + 2M_b)}{2 (M_r + M_b)}$$

$$\Rightarrow P = 2\pi \sqrt{\frac{(M_b + \frac{1}{3} M_r) L^2}{g (M_r + M_b) \frac{L (M_r + 2M_b)}{2 (M_r + M_b)}}} \quad 20 \\ = 2\pi \sqrt{\frac{2L}{3g} \frac{(3M_b + M_r)}{(M_r + 2M_b)}}$$

rod: $M_r = M, M_b = 0$

$$\Rightarrow P = 2\pi \sqrt{\frac{2L}{3g}}$$

$\hookrightarrow$ agrees with solution to b.)

Bob: $M_b = M, M_r = 0$

$$\Rightarrow P = 2\pi \sqrt{\frac{L}{g}}$$

$\hookrightarrow$ agrees with solution to a.)

For the remainder of this problem you can consider an ideal pendulum, for which you will calculate the effect of changing its environment.

Most substances expand with increasing temperature, and so a metal rod will expand by a fraction $\alpha\delta T$ if the temperature is changed by δT , where α is the coefficient of linear expansion.

- (d) Consider the effect of this expansion on a pendulum clock with a pendulum made from brass, with $\alpha = 19 \times 10^{-6} \text{ K}^{-1}$. What temperature change can this clock tolerate if it is to remain accurate to 1 second in 24 hours? [5]

- Let $L_0 = \text{initial length}$ and $P_0 = \text{initial period}$

$$P_0 = 2\pi\sqrt{\frac{L_0}{g}}$$

- Due to expansion $L = L_0(1 + \alpha\delta T)$

- Then the change in the period is:

$$\begin{aligned}\delta P &= 2\pi\sqrt{\frac{L}{g}} - 2\pi\sqrt{\frac{L_0}{g}} \\ &= 2\pi\sqrt{\frac{L_0(1 + \alpha\delta T)}{g}} - 2\pi\sqrt{\frac{L_0}{g}} \\ &= 2\pi\sqrt{\frac{L_0}{g}} \left(\sqrt{1 + \alpha\delta T} - 1 \right)\end{aligned}$$

• Taylor expanding the square-root as $\alpha\delta T \ll 1$,

$$\approx 2\pi\sqrt{\frac{L_0}{g}} \times \frac{\alpha\delta T}{2}$$

$$\Rightarrow \frac{\delta P}{P_0} = \frac{\alpha\delta T}{2}$$

$$\begin{aligned}\Rightarrow \delta T &= \frac{2\delta P}{\alpha P_0} \\ &= \frac{2 \times 1}{19 \times 10^{-6} \times 24 \times 60 \times 60} \\ &= 1.22 \text{ K}\end{aligned}$$

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$$\Rightarrow \boxed{\delta T = 1.2 \text{ K}}$$

- (e) Repeat the calculation in part (d) for a pendulum made from Invar alloy, with $\alpha = 1.2 \times 10^{-6} \text{ K}^{-1}$. [2]

$$\delta T = \frac{2 \times 1}{1.2 \times 10^{-6} \times 24 \times 60 \times 60}$$
$$= 19.3 \text{ K}$$

$$\Rightarrow \boxed{\delta T = 19 \text{ K}}$$