

Problem Set 3: Skills for University Physics

Issued on 17th September 2026, presented at webinars on the 16th and 30th September -
written by Leo Ball

1. ENGAA, Paper 1, 2017

A fruit stall sells apples costing £ x each, and pears costing £ y each.

Sam bought 2 apples and 5 pears, and the total cost of these was £ P .

Lesley bought 3 apples and 2 pears, and the total cost of these was £ Q .

Which of the following is an expression for the cost, in pounds (£), of a pear?

A $\frac{2Q - 3P}{3}$

B $\frac{2Q - 3P}{11}$

C $\frac{Q - P}{3}$

D $\frac{Q - P}{11}$

E $\frac{P - Q}{3}$

F $\frac{3P - 2Q}{3}$

G $\frac{3P - 2Q}{11}$

2. PAT Specimen Paper, 2017

A radioactive sample contains two different isotopes, A and B. A has a half-life of 3 days, and B has a half-life of 6 days. Initially in the sample there are twice as many atoms of A as of B. At what time will the ratio of the number of atoms of A to B be reversed?

A	B	C	D	E
3 days	6 days	9 days	12 days	never

Note: like most questions at this level, this problem can be solved in several ways. Once you've solved it, try to think about an alternative approach.

3. PAT Specimen Paper, 2017

A jar contains buttons of four different colours. There are twice as many yellow as green, twice as many red as yellow, and twice as many blue as red. What is the probability of taking from the jar:

- a) a blue button;
- b) a red button;
- c) a yellow button; or
- d) a green button?

You may assume that you are only taking one button at a time and replacing it in the jar before selecting the next one.

4. Question 4 – BPhO Senior Challenge 2017

A spider of mass m hangs from the end of a single, elastic, thread obeying Hooke's Law, of a web attached to the ceiling of a room. The extension of the thread is equal to the natural length of the thread ℓ_0 , when the spider is at the bottom end of the thread.

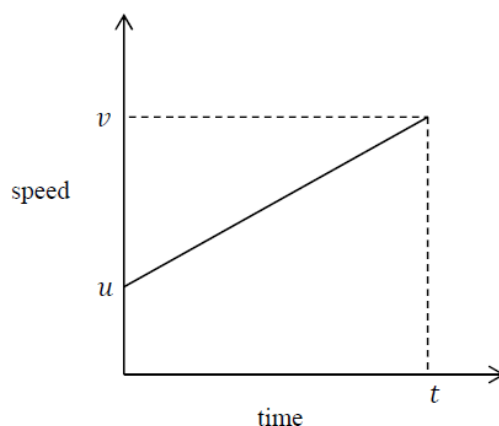
- a) Explain why the work done by the spider in climbing the thread all the way to the ceiling is less than the work done to climb a vertical distance $2\ell_0$ without the thread.
- b) What fraction of the work that would be required to climb to the ceiling does the spider save by using the elastic thread to climb up?

5. **Question 5 – BPhO Senior Challenge 2016**

a) When a particle falls under gravity it falls with a constant acceleration. When the particle has an initial speed u , the speed time graph is shown below. Using this graph, show how the equation of motion

$$s = ut + \frac{1}{2}at^2$$

is obtained, where the symbols have their usual meaning.



Two points A and B are located on a vertical line with point A vertically above point B. A particle is released from rest at A and at the same time another particle is projected vertically upwards from B at velocity v . The particles collide when the top one has fallen a distance y . The height of point A above point B is h .

b) Show that the time, t , from the start of the motion to collision is given by:

$$t = \frac{h}{v}$$

c) How does the distance the particle falls, y , from point A depend upon v , h and g ?

d) Sketch the velocity-time graphs, on the same set of axes, for the two particles. If $h=100\text{m}$, $v=20\text{ms}^{-1}$ and $g=10\text{ms}^{-2}$, mark on the graphs the (t,v) values where the particles collide. Take v to be positive upwards.

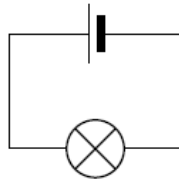
6. PAT 2022

The Trojan asteroids share Jupiter's orbit around the Sun: approximately circular with a mean radius 5.2 AU (1 AU = 1.5×10^{11} m is the mean radius of the Earth's orbit around the Sun). The Trojans are clustered around two points labelled L4 and L5, where the L4 point is 60° ahead of Jupiter in its orbit and the L5 point is 60° behind Jupiter in its orbit.

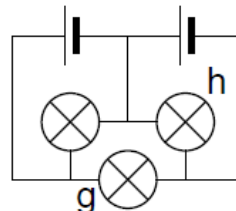
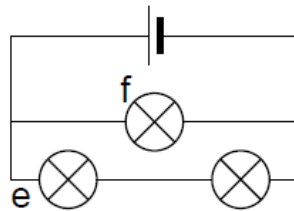
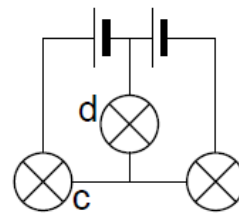
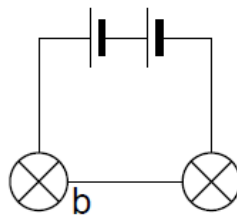
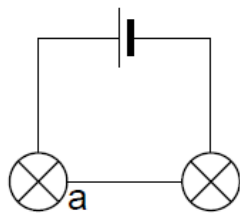
- a) Determine the mean distance between the asteroids 588 Achilles (at the L4 point) and 617 Patroclus (at the L5 point).
- b) A spacecraft travels in a straight line between the two asteroids, accelerating at 10 ms^{-2} until the half-way point between the asteroids, and decelerating at 10 ms^{-2} from there to the endpoint. Assuming that the asteroids are approximately stationary on the timescale of the journey and neglecting any gravitational effects of Jupiter or the Sun, find the total travel time.
- c) Explain why the assumption that the asteroids are approximately stationary during the journey is well-justified.

7. PAT 2006

The diagram below shows a circuit in which the bulb lights up with normal brightness.



In the circuits below the bulbs and cells all have the same specifications as the bulb and cell shown above.



Determine whether the bulbs marked by letters in these circuits are brighter than normal, normal, dimmer than normal, or off. [8]

8. PAT 2007

The current in amperes through a certain type of non-linear resistor is given by $I = 0.05V^3$, where V is the potential difference in volts across the resistor.

This resistor is connected in series to a fixed resistor and a constant voltage source of $9V$ is connected across the series combination. What value of resistance should the fixed resistor have so that a current of $0.40A$ flows?

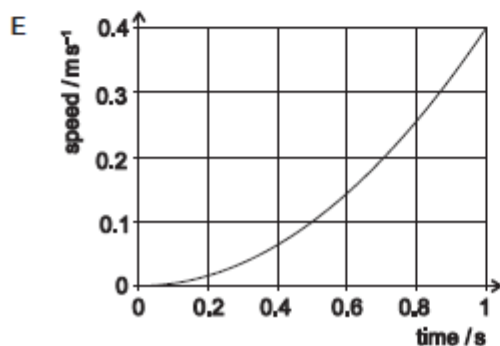
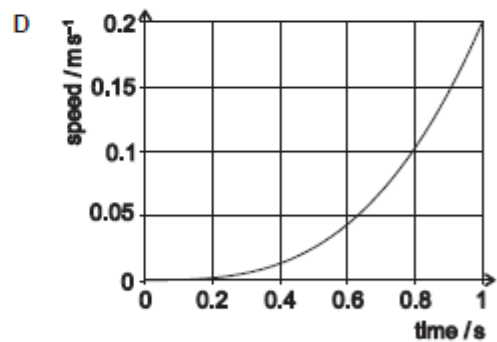
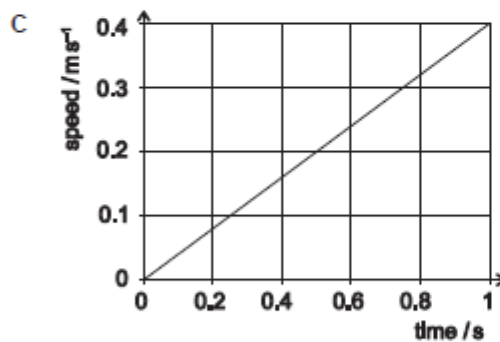
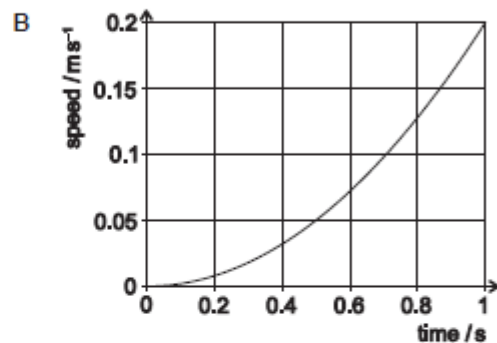
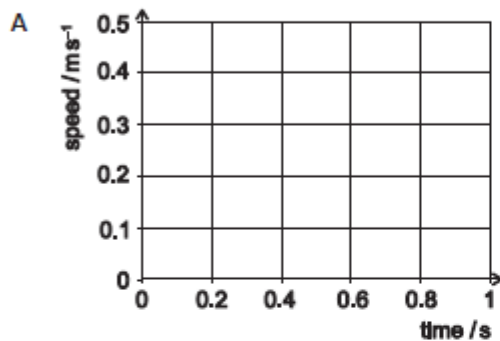
9. ENGAA 2017 Paper 2

A ball of mass $m = 0.5 \text{ kg}$ is at rest a distance d above the flat floor of a spacecraft.

Installed in the floor is an artificial gravity generator which produces a field at right angles to the floor, directed towards the floor. There is no air in the spacecraft.

The generator is switched on at time $t = 0 \text{ s}$ and produces a field g that increases linearly with time, such that $g = 0.4 t \text{ m s}^{-2}$. The artificial gravity is the only force experienced by the ball.

- a) Assuming that the ball does not hit the floor within the first second of motion, which of these graphs represents the speed of the ball plotted against time? [2 marks]



Try using calculus to solve this problem.

b) Which of these expressions gives the time taken for the ball to first hit the floor? [2 marks]

A $(15d)^{\frac{1}{3}}$

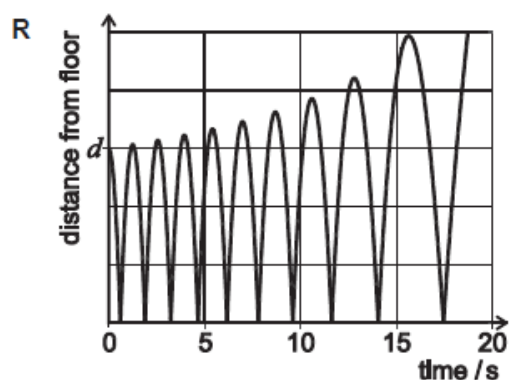
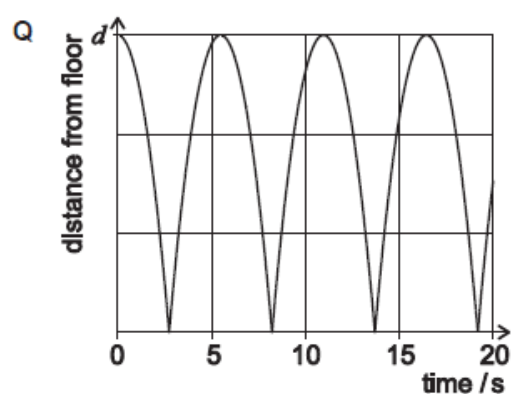
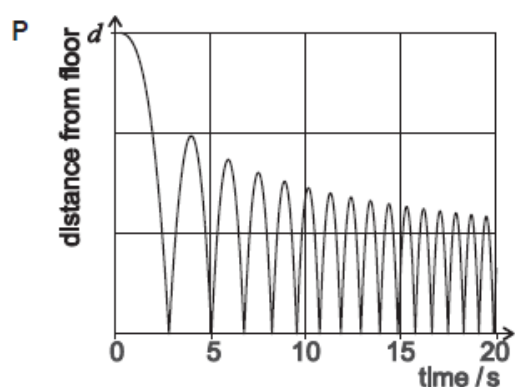
B $(5d)^{\frac{1}{3}}$

C $(5d)^{\frac{1}{2}}$

D $\left(\frac{15d}{2}\right)^{\frac{1}{3}}$

E $\left(\frac{5d}{2}\right)^{\frac{1}{3}}$

c) The ball bounces and hits the floor repeatedly. Which of these graphs might represent the position of the ball plotted against time? [3 marks]



- A P only
- B Q only
- C R only
- D P and Q only
- E Q and R only

- d) Force is usually measured in Newtons (N). Given that $F=ma$, which of the following is an alternative unit for force? [1 mark]

- A kg s m^{-2}
- B $\text{kg}^{-1} \text{m}^{-1} \text{s}^2$
- C kg m s^{-2}
- D $\text{N kg}^{-1} \text{m}^{-1} \text{s}^2$
- E $\text{N}^{-1} \text{kg m s}^{-2}$

- e) Air is now injected into the spacecraft, creating air resistance. The drag force D on the ball is given by

$$D = \frac{1}{2} X \rho v^2 A$$

where ρ is the air density, v is the ball's speed, A is its cross-sectional area and X is an unknown parameter.

What are the units of X ?

[2 marks]

- A m s^{-2}
- B m s^{-1}
- C $\text{kg}^{-1} \text{m}^{-1} \text{s}^2$
- D kg m s^{-2}
- E X has no units

10. Question 10 - PAT 2022

Suppose $f(t) = 4t$ and $g(x) = \frac{3}{2} (3x - x^2)$. Consider the inequality

$$\frac{dg(x)}{dx} > \int_{3/2}^x f(t) dt.$$

For which values of x is this inequality satisfied?

11. BPhO Senior Challenge 2016

Interplanetary satellites are very complex platforms with dozens of scientific instruments, mechanical devices and radio transmitters and receivers on board. They require considerable power and operate over many years, and those travelling to the outer planets cannot use solar power. They rely instead on Radioisotope Thermal Generators (RTG), which produces heat by simple radioactive decay, and this heat is converted to electrical energy by heating one side of a semiconductor and cooling the other. There are no moving parts and the efficiency is low at typically 8%, but the reliability is very high.

Rather than have one large mass of the most commonly used radioactive isotope, plutonium-238, many small pellets are used to generate the energy required to run the satellite's systems. One pellet is used to produce 5 W of electrical power.

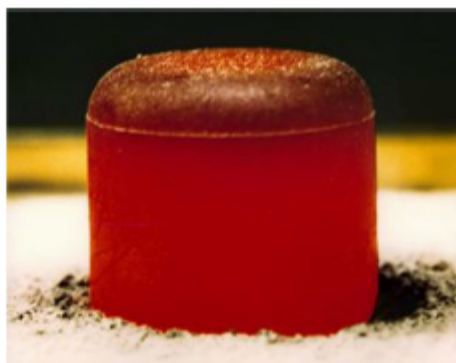


Figure 6. An image of a marshmallow-sized Pu-238 pellet glowing red hot.

(Source: Wikimedia Commons, attributed to Los Alamos National Laboratory)

The following information has been sourced from the internet, and one can find a picture of the pellet, which is described as the size of a marshmallow. Use the following information to calculate the volume of such a pellet.

Use the units as a help in your calculations. State what you intend to calculate even if you do not know how to proceed.

DATA:

Efficiency of conversion of thermal power to electrical 8%

Energy released in a single alpha decay 5.5 MeV

1 MeV = 1.6×10^{-13} J

Avogadro's number, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

Half-life of ^{238}Pu is 87.4 years

When the isotope is new and little of it has decayed, the activity, A_0 (i.e. the number of decays per second) is equal to $0.693 \times$ the number of radioactive atoms present, N_0

$\div$ half life in seconds, t_{hl} . i.e.

$$A_0 = \frac{0.693 \times N_0}{t_{hl}}$$

The plutonium is supplied as a ceramic pellet of PuO_2 which has a density of 10 g cm^{-3}

The mass number of Pu-238 is 238

The mass number of oxygen (O) is 16

Only 80% of the plutonium atoms are actually radioactive Pu-238

12. PAT 2006

This problem concerns the mathematical treatment of a simple model of an electric toy car of mass m , which is initially stationary. The batteries in the car can be considered as an electrical power source with constant power P . You may neglect air-resistance and other experimental imperfections in the calculations.

- (a) Suppose that the car is placed on a level surface and that the motor can be treated as a device which converts electrical energy directly into kinetic energy. Calculate the kinetic energy of the car as a function of time, and hence its velocity as a function of time. [3]
- (b) Hence calculate the acceleration of the car and the distance traveled by the car as a function of time. [4]
- (c) Find the limiting value of the velocity at very large times and comment on whether your result seems reasonable. [2]
- (d) Find the limiting value of the acceleration at very large and very small times and comment on whether your results seem reasonable. [4]
- (e) Instead of driving the car forward the motor could be used to lift the car up a vertical rope. Calculate the height which the car can reach as a function of time, and hence calculate the velocity at which it climbs the rope. [3]
- (f) The calculation in part (e) ignored the fact that as the car is moving then some of the motor's power must be used to give kinetic energy to the car. Assuming that the approach you used in part (e) is correct, calculate the ratio between the kinetic energy and the potential energy of the car climbing the rope as a function of time. Use your result to determine under what circumstances it is reasonable to ignore kinetic energy in this way. [4]

13. PAT 2015

Using your knowledge of solar eclipses, estimate the radius of the Moon. You may assume the radius of the Sun is 700; 000 km, the distance from the Earth to the Sun is 150; 000; 000 km, and the distance from the Earth to the centre of the Moon is 400; 000 km **[3]**

14. PAT 2015

A non-rotating ball of mass 2 kg slides on a smooth, frictionless, horizontal surface at a speed of 1 m/s. It collides elastically and head-on with a stationary ball of mass 1 kg. What are the speeds of the two balls after the collision?

15. PAT 2015

A garden hose with a cross-sectional area **A** ejects water at a rate of **x**, usually measured in units of m^3s^{-1} .

- a) What is the speed of the water leaving the nozzle?
- b) The water hits a wall close to the end of the garden hose, perpendicular to the direction of flow. What is the force on the wall if:
 - i. the water falls to the ground when it hits the wall, or
 - ii. the water rebounds horizontally?

You may assume the density of water is **ρ** , usually measured in units of kgm^{-3} .

16. NSAA 2016 Section 1

A square PQRS is drawn above the x -axis with the side PQ on the x -axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x + 6y + 4 = 0$

B $x^2 + y^2 - 4x + 6y + 9 = 0$

C $x^2 + y^2 + 4x - 6y + 4 = 0$

D $x^2 + y^2 + 4x - 6y + 9 = 0$

E $x^2 + y^2 - 6x - 4y + 9 = 0$

F $x^2 + y^2 - 6x + 4y + 4 = 0$

G $x^2 + y^2 + 6x - 4y + 4 = 0$

H $x^2 + y^2 + 6x + 4y + 9 = 0$

17. NSAA 2016 Section 1

Given that $7\cos \theta - 3\tan \theta \sin \theta = 1$, which one of the following is true?

A $\cos \theta = -\frac{3}{5}$ or $-\frac{1}{2}$

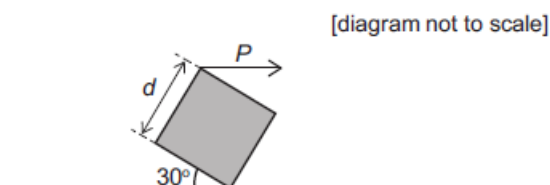
B $\cos \theta = -\frac{3}{5}$ or $\frac{1}{2}$

C $\cos \theta = \frac{3}{5}$ or $\frac{1}{2}$

D $\cos \theta = \frac{3}{5}$ or $-\frac{1}{2}$

18. NSAA 2016 Section 1

The diagram shows a uniform, solid, heavy cube with side d . The cube rests with one of its edges in contact with a table that is perfectly level. A horizontal force P acts on another edge of the cube, and the cube is stationary.



Below are four statements about the forces on the cube.

- 1 It is possible that there is no frictional force between the cube and the table.
- 2 There must be a frictional force acting to the left between the cube and the table.
- 3 There must be a frictional force acting to the right between the cube and the table.
- 4 Force P has a clockwise moment about the edge in contact with the table equal to $P \times d$.

Which of the statements is/are correct?

- A 1 only
- B 2 only
- C 3 only
- D 1 and 4 only
- E 2 and 4 only
- F 3 and 4 only

19. NSAA 2016 Section 1

An object is fired vertically upwards from the ground at time $t = 0$ s in still air at a speed of 8.0 m s^{-1} .

On the way up, what is the height of the object above the ground when it has a speed of 2.0 m s^{-1} , and at what time does it reach this height on the way down?

(The gravitational field strength g is 10 N kg^{-1} . Air resistance can be ignored.)

	<i>height / m</i>	<i>time / s</i>
A	2.4	0.60
B	2.4	0.64
C	2.4	1.0
D	2.4	2.0
E	3.0	0.60
F	3.0	0.64
G	3.0	1.0
H	3.0	2.0

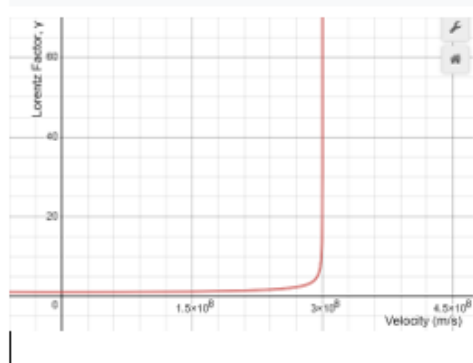
20. BPhO Senior Challenge 2024

The Lorentz factor, γ , is derived from Dutch physicist Henrik Lorentz's work on electrodynamics. It appears in Einstein's Special Relativity equations and describes the observed change in the physical characteristics of a body (length, time, mass, momentum, and energy) which is moving relative to an observer.

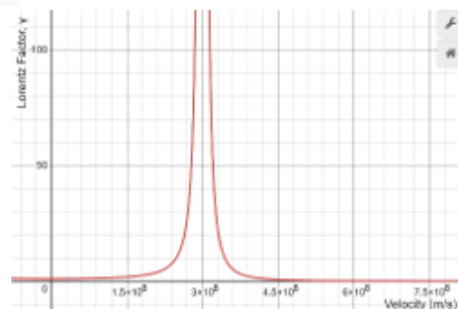
The equation for γ is commonly written as $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the velocity of the moving object and c is the speed of light ($3 \times 10^8 \text{ ms}^{-1}$).

Which of the following graphs represents γ as a function of v ?

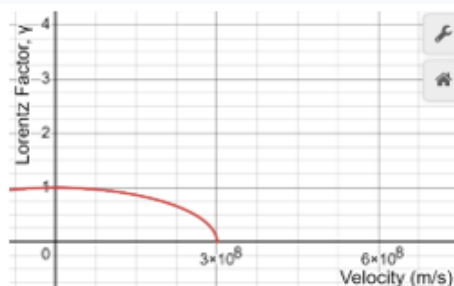
A



B



C



D

