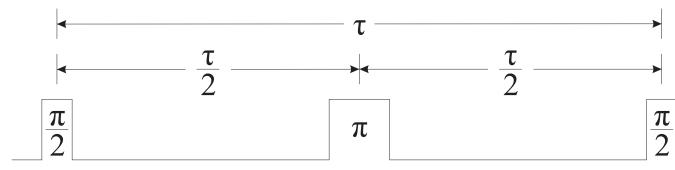


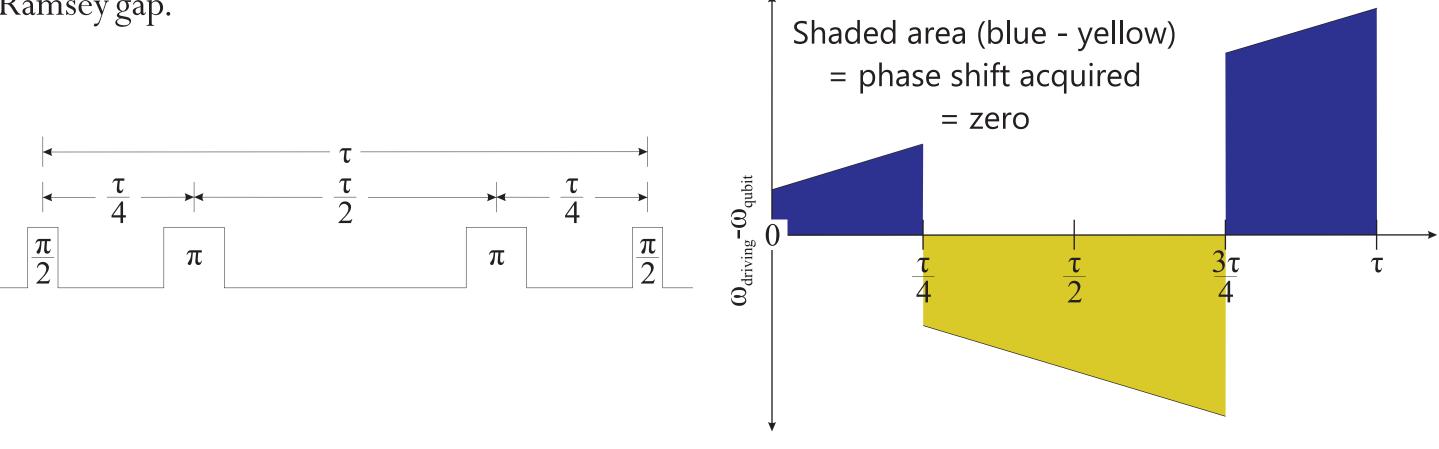


Generalised Spin-Echo to Protect Against Changing Magnetic Field

The Hahn spin-echo is a well-known method to improve the coherence of a Ramsey experiment, if the frequency of the atom's free precession is slightly different to the driving radiation. A p-pulse in the middle of the gap can "unwind" the excess phase acquired from this offset, and restore the fringe contrast.



Any variation in the frequency error, such as a drifting magnetic field, will cause an uncorrected error. However, we see by inspection that *two* p-pulses can precisely cancel out the error if the frequency is varying linearly with time. They must be placed at times $t = \tau/4$ and $t = 3\tau/4$, where τ is the total Ramsey gap.



To generalise further, suppose the detuning $\delta(t)$ is an (n-1)th order polynomial with time: $\delta(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_{n-1} t^{n-1}.$ Perfect, instantaneous π -pulses occur at times $\alpha_1 \tau, \alpha_2 \tau, \ldots, \alpha_n \tau$. The spurious accumulated phase ϕ_{err} is given by integrating the detuning with respect to time, so demanding that ϕ_{err} vanishes requires us to solve:

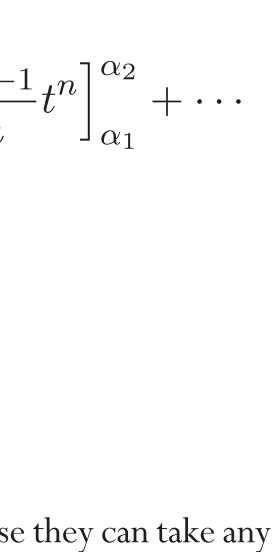
$$\begin{aligned} 0 &= \phi_{err} \\ &= \int_0^{\alpha_1} \delta(t) \, \mathrm{d}t - \int_{\alpha_1}^{\alpha_2} \delta(t) \, \mathrm{d}t + \dots + (-1)^n \int_{\alpha_n}^1 \delta(t) \, \mathrm{d}t \\ &= \left[a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_0^{\alpha_1} - \left[a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_{\alpha_n}^1 \\ &+ (-1)^n \left[a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_{\alpha_n}^1 \\ &= \sum_{j=0}^{n-1} \frac{a_j}{j+1} \left((-1)^n - 2 \sum_{i=1}^n (-1)^i \alpha_i^j \right) \end{aligned}$$

This equation must be independently true for each polynomial coefficient *a_i*, because they can take any (real) value. So we obtain a set of simultaneous equations:

$$(-1)^n - 2\sum_{i=1}^n (-1)^i \alpha_i^j = 0 \quad \forall j = 1, 2, \dots, n.$$

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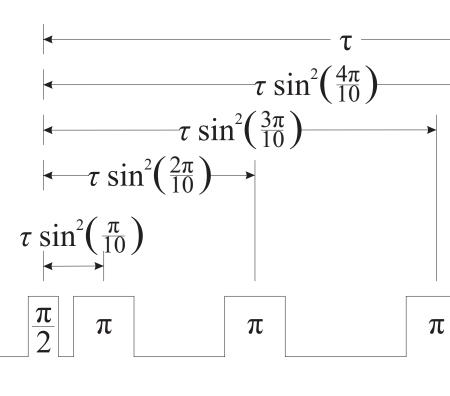
SCWebster DTCAllcock AHBurrell MJCurtis NMLinke JASherman DMLucas D N Stacey A M Steane Dynamical Decoupling of a ⁴³Ca⁺ Memory Qubit Ion Trap Quantum Computing Group - Department of Physics - University of Oxford



These simultaneous equations are solved when the pulse times α_i take the values:

 $\alpha_i = \sin^2 \left(\frac{\pi}{2} \frac{i}{n+1} \right)$

This pulse sequence is illustrated below for n = 4.

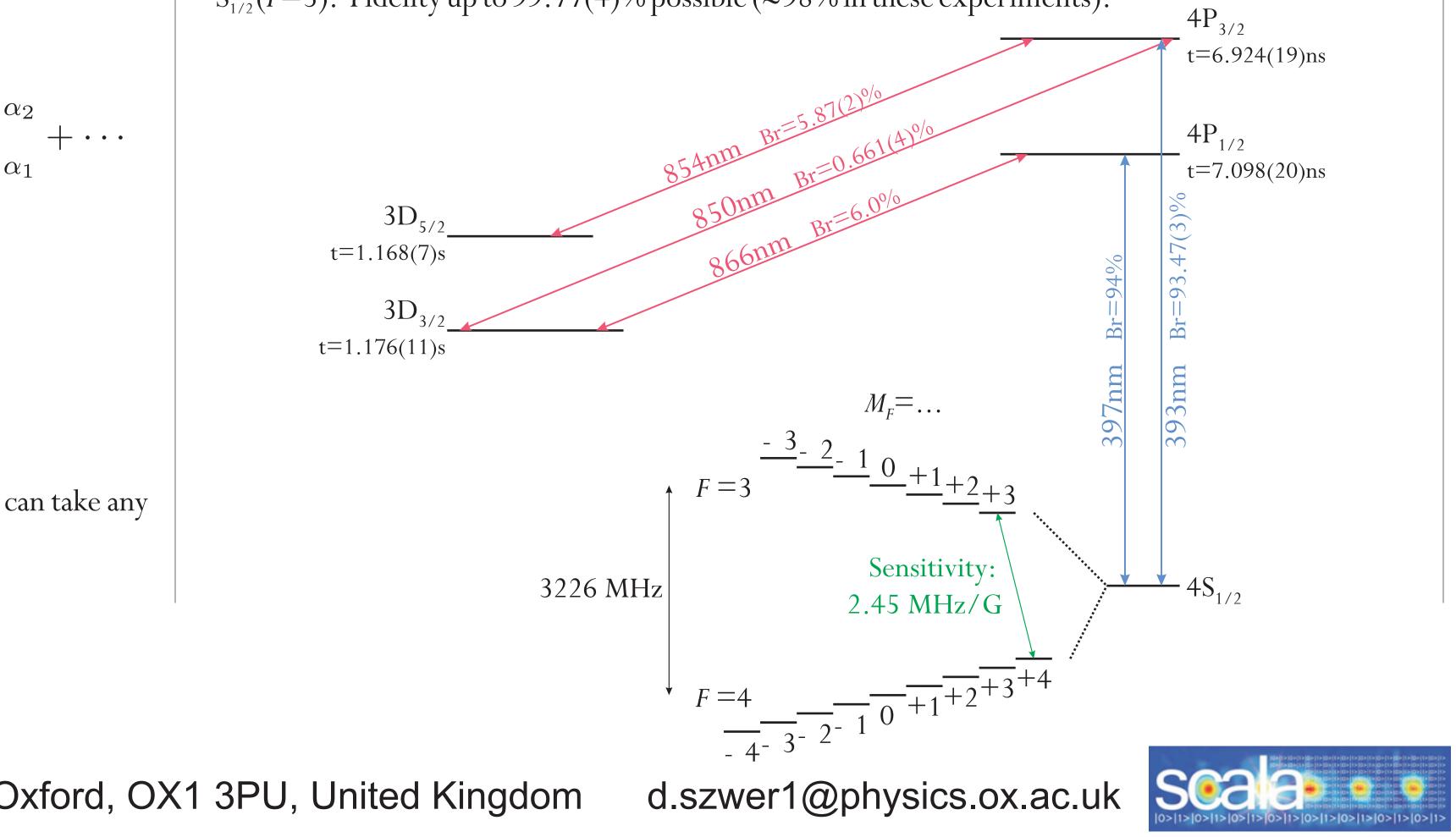


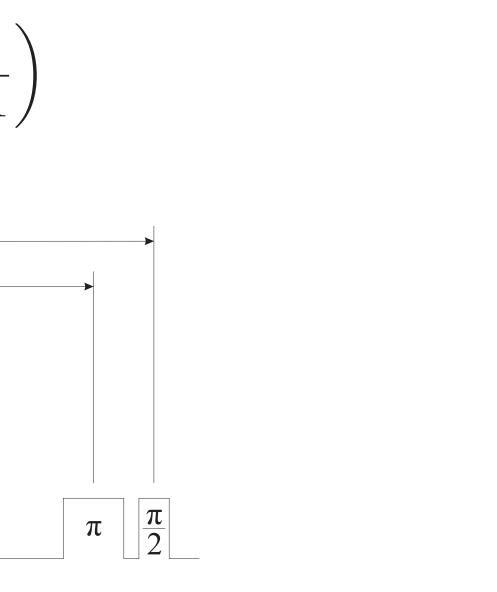
An *n* pulse sequence precisely cancels out the spurious accumulated phase when the detuning is an (n-1)th order polynomial in time.

This sequence is called Uhrig Dynamical Decoupling (UDD). Previously discovered in a solid-state context, by considering a spin-echo sequence as a frequency domain filter. See: "Exact results on dynamical decoupling by p pulses in quantum information processes", Götz S Uhrig, New Journal of Physics **10** (2008) 083024. Biercuk *et al.* have implemented UDD on an ensemble of ions in a Penning trap: "Experimental Uhrig dynamical decoupling using trapped ions", Michael J Biercuk et al., Physical Review A **79** (2009) 062324.

⁴³Ca⁺ Hyperfine Qubit

- Qubit stored in $S_{1/2}$ hyperfine-split level: $(F=3, M_F=+3)$ and $(F=4, M_F=+4)$ states.
- Sensitive to magnetic field: 2.45 MHz/gauss. 3.226 GHz at zero field. Work at 2.4 G. Rabi frequency up to $18 \text{ kHz} (t_{\pi} \approx 28 \mu \text{s})$.
- Doppler cooling, and observation of fluorescence, at 397 nm. Repump at 866 nm. • Readout: use 393 nm to shelve ion in $D_{5/2}$, with 850 nm to repump from $D_{3/2}$. Frequency selective -
- shelve only from $S_{1/2}(F=4)$. Then apply 397 nm + 866 nm and observe fluorescence only if ion was in $S_{1/2}$ (*F*=3). Fidelity up to 99.77(4)% possible (\approx 98% in these experiments).





Experimental Results

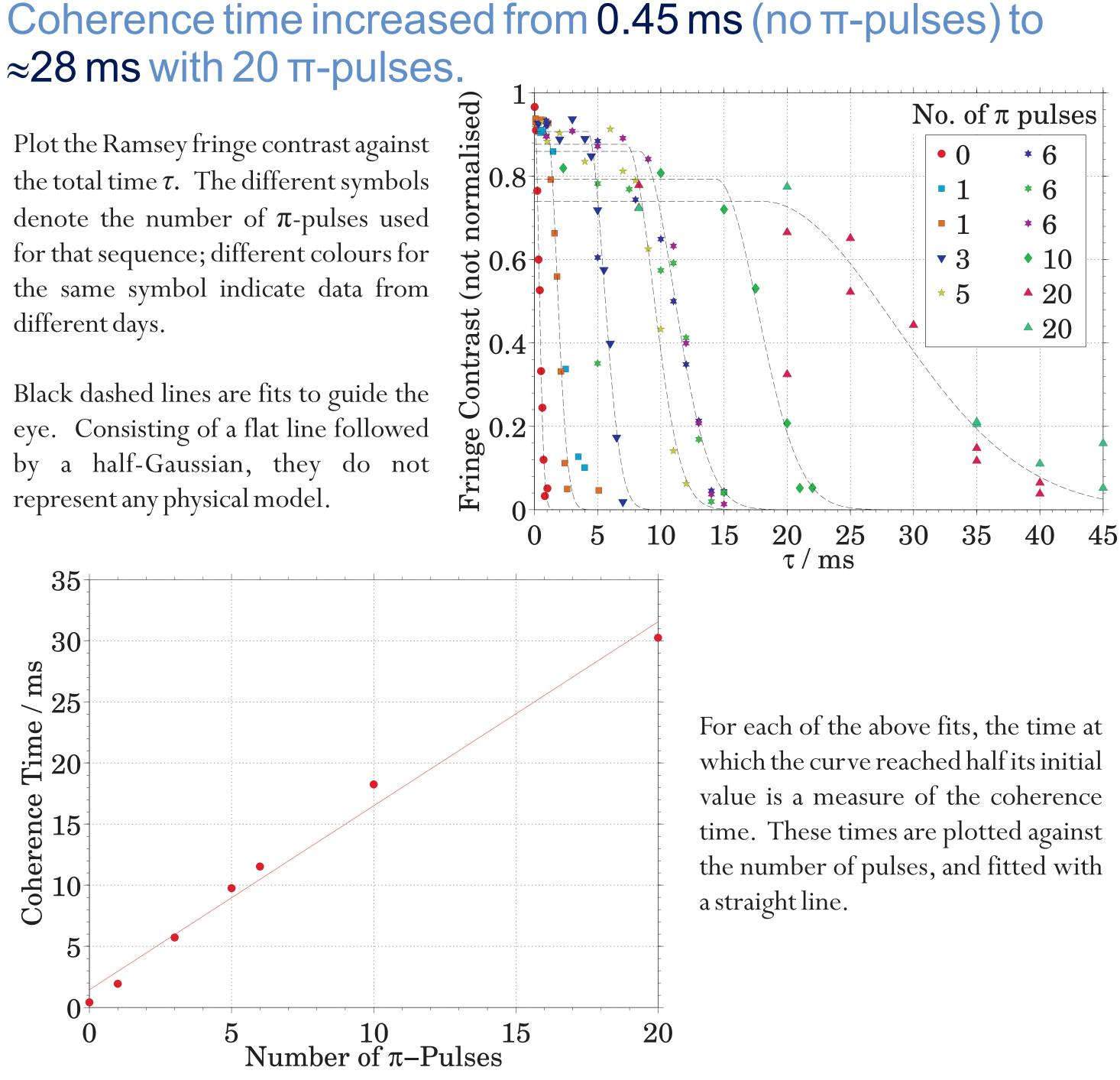
We have implemented Dynamical Decoupling using the ${}^{43}Ca^+$ hyperfine qubit.

397 o +	Prepare $S_{1/2}$ (<i>F</i> =4, M_F =+4)		Ramsey Experiment with <i>n</i> UDD p-pulses (scan phase of final p / 2). τ and <i>n</i> varied, and fringe contrast recorded.		Shelve	Detect fluorescence
866 Repump						
3.22 GHz Microwaves		₽ <u>⁄2</u> p	τp	p ½		
393 σ + Shelve			$\rightarrow t_{\pi} \approx 28 \mu s$			
850 σ + Repump						
397 Doppler						
854 Deshelve						
	I			I		I

different days.

eye. Consisting of a flat line followed by a half-Gaussian, they do not represent any physical model.

EPSR



UDD could also be performed using the magnetic-field insensitive $M_F = 0$ "clock" states. Previously, we used a single spin-echo pulse on such a qubit and observed negligible (d1%) decoherence in 1 s. Dynamical decoupling could extend this even further. See: "A long-lived memory qubit on a lowdecoherence quantum bus", David M Lucas et al., arXiv:0710.4421 [quant-ph].





