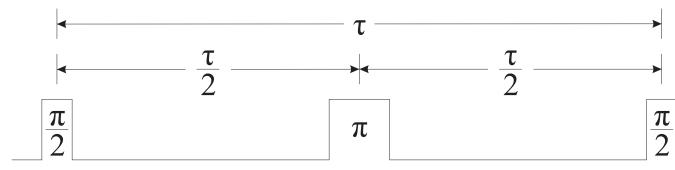


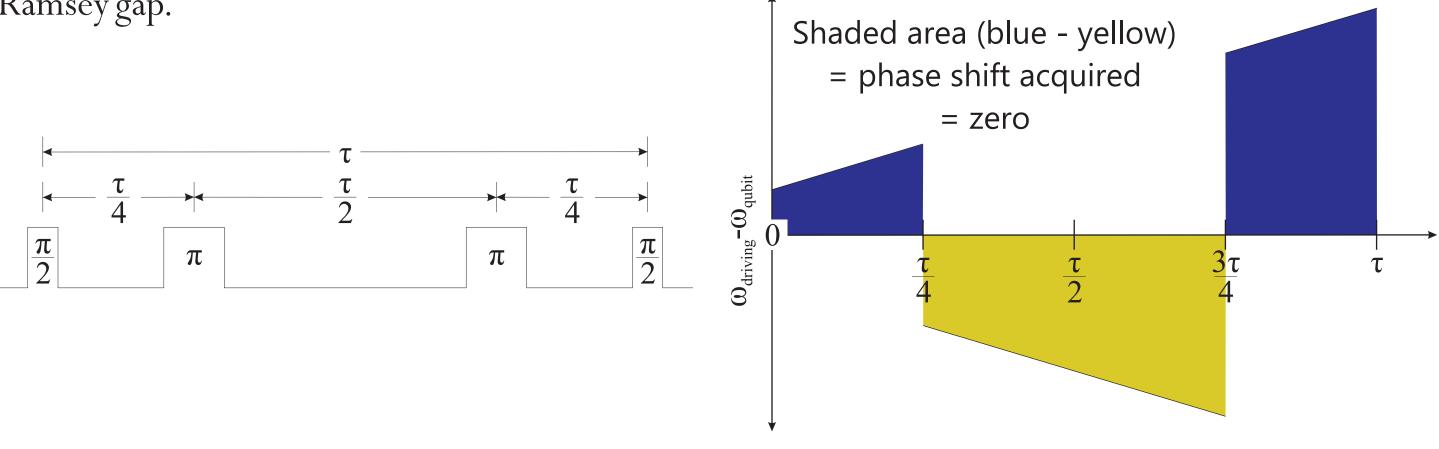


# Generalised Spin-Echo to Protect Against Changing Magnetic Field

The Hahn spin-echo is a well-known method to improve the coherence of a Ramsey experiment, if the frequency of the atom's free precession is slightly different to the driving radiation. A p-pulse in the middle of the gap can "unwind" the excess phase acquired from this offset, and restore the fringe contrast.



Any variation in the frequency error, such as a drifting magnetic field, will cause an uncorrected error. However, we see by inspection that *two* p-pulses can precisely cancel out the error if the frequency is varying linearly with time. They must be placed at times  $t = \tau/4$  and  $t = 3\tau/4$ , where  $\tau$  is the total Ramsey gap.



To generalise further, suppose the detuning  $\delta(t)$  is an (n-1)th order polynomial with time:  $\delta(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_{n-1} t^{n-1}.$  Perfect, instantaneous  $\pi$ -pulses occur at times  $\alpha_1 \tau, \alpha_2 \tau, \ldots, \alpha_n \tau$ . The spurious accumulated phase  $\phi_{err}$  is given by integrating the detuning with respect to time, so demanding that  $\phi_{err}$  vanishes requires us to solve:

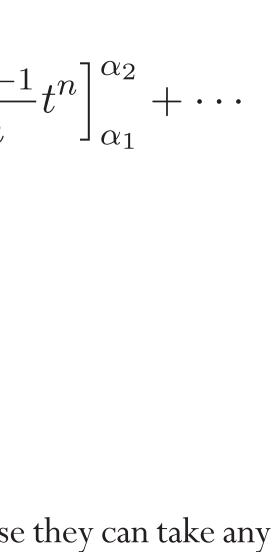
$$\begin{aligned} 0 &= \phi_{err} \\ &= \int_0^{\alpha_1} \delta(t) \, \mathrm{d}t - \int_{\alpha_1}^{\alpha_2} \delta(t) \, \mathrm{d}t + \dots + (-1)^n \int_{\alpha_n}^1 \delta(t) \, \mathrm{d}t \\ &= \left[ a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_0^{\alpha_1} - \left[ a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_{\alpha_n}^1 \\ &+ (-1)^n \left[ a_0 t + \frac{a_1}{2} t^2 + \dots + \frac{a_{n-1}}{n} t^n \right]_{\alpha_n}^1 \\ &= \sum_{j=0}^{n-1} \frac{a_j}{j+1} \left( (-1)^n - 2 \sum_{i=1}^n (-1)^i \alpha_i^j \right) \end{aligned}$$

This equation must be independently true for each polynomial coefficient *a<sub>i</sub>*, because they can take any (real) value. So we obtain a set of simultaneous equations:

$$(-1)^n - 2\sum_{i=1}^n (-1)^i \alpha_i^j = 0 \quad \forall j = 1, 2, \dots, n.$$

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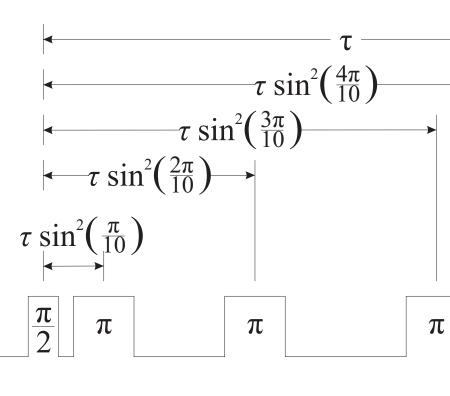
## SCWebster DTCAllcock AHBurrell MJCurtis NMLinke JASherman DMLucas D N Stacey A M Steane Dynamical Decoupling of a <sup>43</sup>Ca<sup>+</sup> Memory Qubit Ion Trap Quantum Computing Group - Department of Physics - University of Oxford



These simultaneous equations are solved when the pulse times  $\alpha_i$  take the values:

 $\alpha_i = \sin^2 \left( \frac{\pi}{2} \frac{i}{n+1} \right)$ 

This pulse sequence is illustrated below for n = 4.

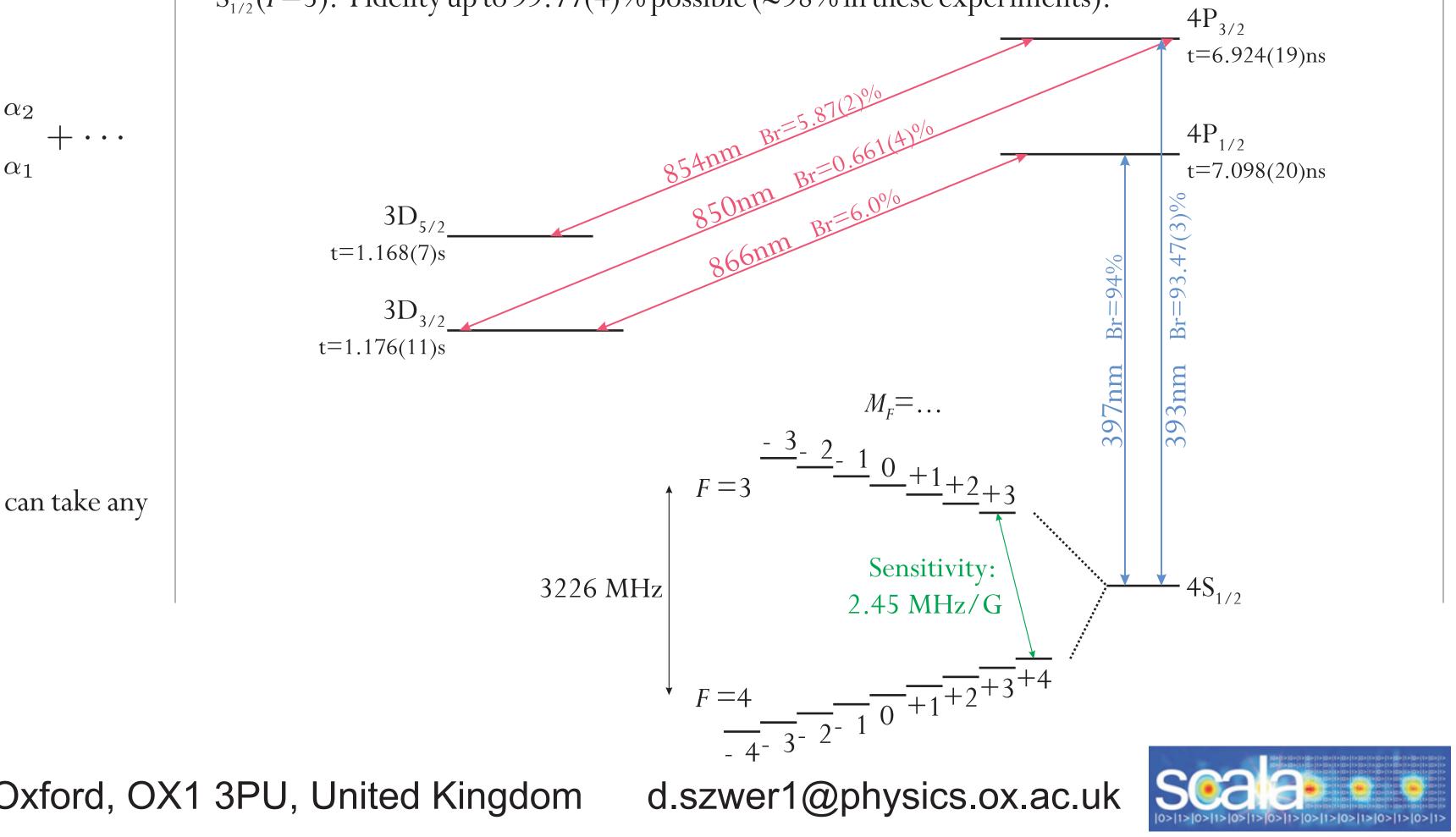


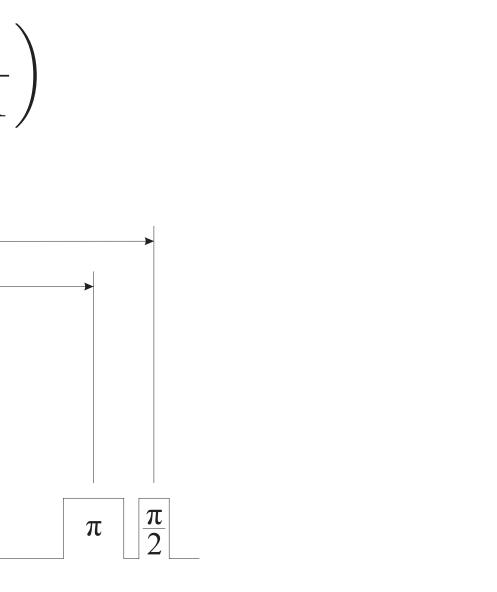
### An *n* pulse sequence precisely cancels out the spurious accumulated phase when the detuning is an (n-1)th order polynomial in time.

This sequence is called Uhrig Dynamical Decoupling (UDD). Previously discovered in a solid-state context, by considering a spin-echo sequence as a frequency domain filter. See: "Exact results on dynamical decoupling by p pulses in quantum information processes", Götz S Uhrig, New Journal of Physics **10** (2008) 083024. Biercuk *et al.* have implemented UDD on an ensemble of ions in a Penning trap: "Experimental Uhrig dynamical decoupling using trapped ions", Michael J Biercuk et al., Physical Review A **79** (2009) 062324.

# <sup>43</sup>Ca<sup>+</sup> Hyperfine Qubit

- Qubit stored in  $S_{1/2}$  hyperfine-split level:  $(F=3, M_F=+3)$  and  $(F=4, M_F=+4)$  states.
- Sensitive to magnetic field: 2.45 MHz/gauss. 3.226 GHz at zero field. Work at 2.4 G. Rabi frequency up to  $18 \text{ kHz} (t_{\pi} \approx 28 \mu \text{s})$ .
- Doppler cooling, and observation of fluorescence, at 397 nm. Repump at 866 nm. • Readout: use 393 nm to shelve ion in  $D_{5/2}$ , with 850 nm to repump from  $D_{3/2}$ . Frequency selective -
- shelve only from  $S_{1/2}(F=4)$ . Then apply 397 nm + 866 nm and observe fluorescence only if ion was in  $S_{1/2}$  (*F*=3). Fidelity up to 99.77(4)% possible ( $\approx$ 98% in these experiments).





## **Experimental Results**

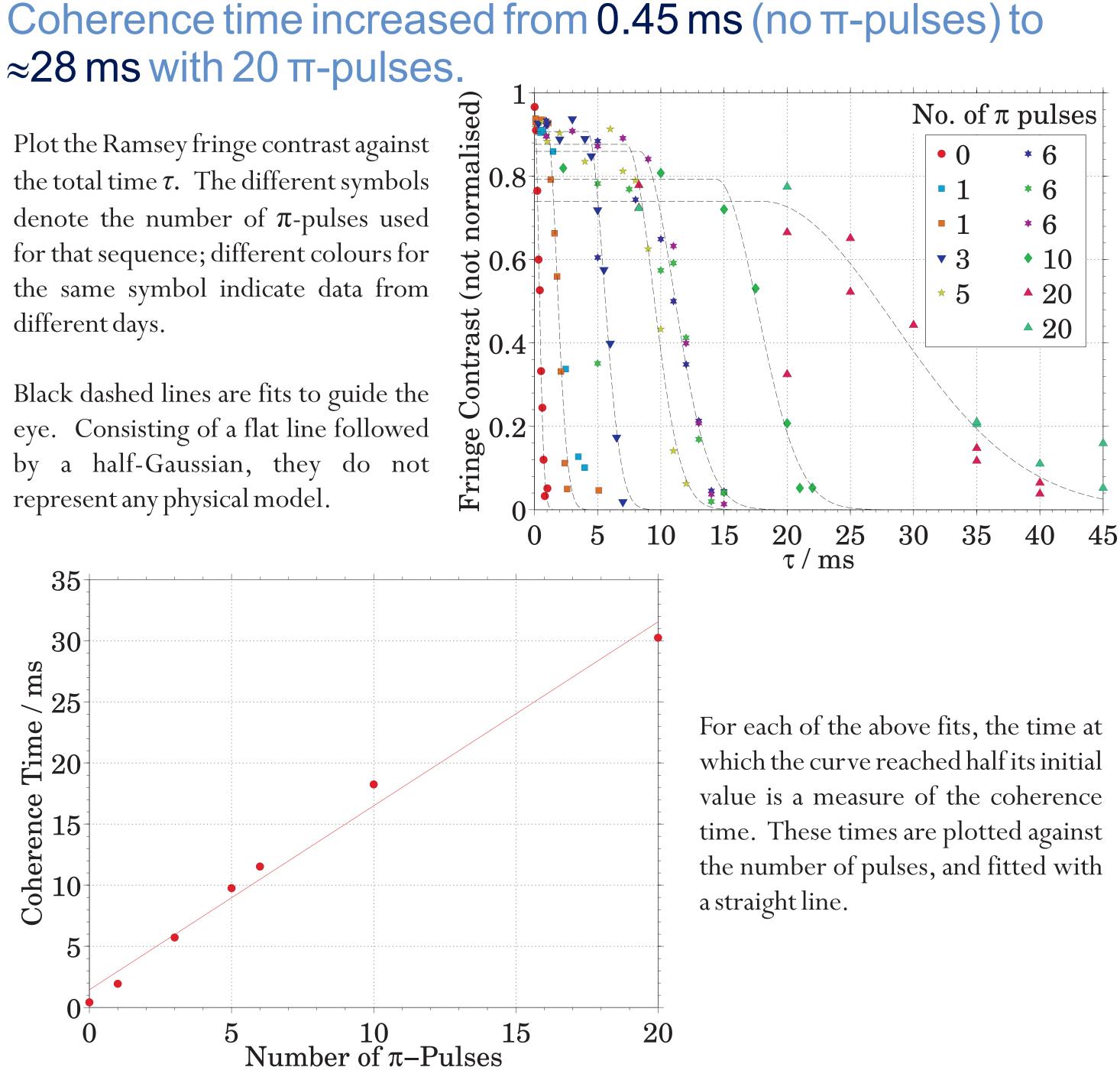
We have implemented Dynamical Decoupling using the  ${}^{43}Ca^+$  hyperfine qubit.

397 <b>o</b> +	Prepare $S_{1/2}$ ( <i>F</i> =4, $M_F$ =+4)		Ramsey Experiment with <i>n</i> UDD p-pulses (scan phase of final p / 2). τ and <i>n</i> varied, and fringe contrast recorded.		Shelve	Detect fluorescence
866 Repump						
3.22 GHz Microwaves		₽ <u>⁄2</u> p	τp	p ½		
393 $\sigma$ + Shelve			$\rightarrow t_{\pi} \approx 28 \mu s$			
850 <b>σ</b> + Repump						
397 Doppler						
854 Deshelve						
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different days.

eye. Consisting of a flat line followed by a half-Gaussian, they do not represent any physical model.

EPSR



UDD could also be performed using the magnetic-field insensitive  $M_F = 0$  "clock" states. Previously, we used a single spin-echo pulse on such a qubit and observed negligible (d1%) decoherence in 1 s. Dynamical decoupling could extend this even further. See: "A long-lived memory qubit on a lowdecoherence quantum bus", David M Lucas et al., arXiv:0710.4421 [quant-ph].





