Deterministic Entanglement and Schrödinger Cat States of Trapped-Ion Spin-Qubits

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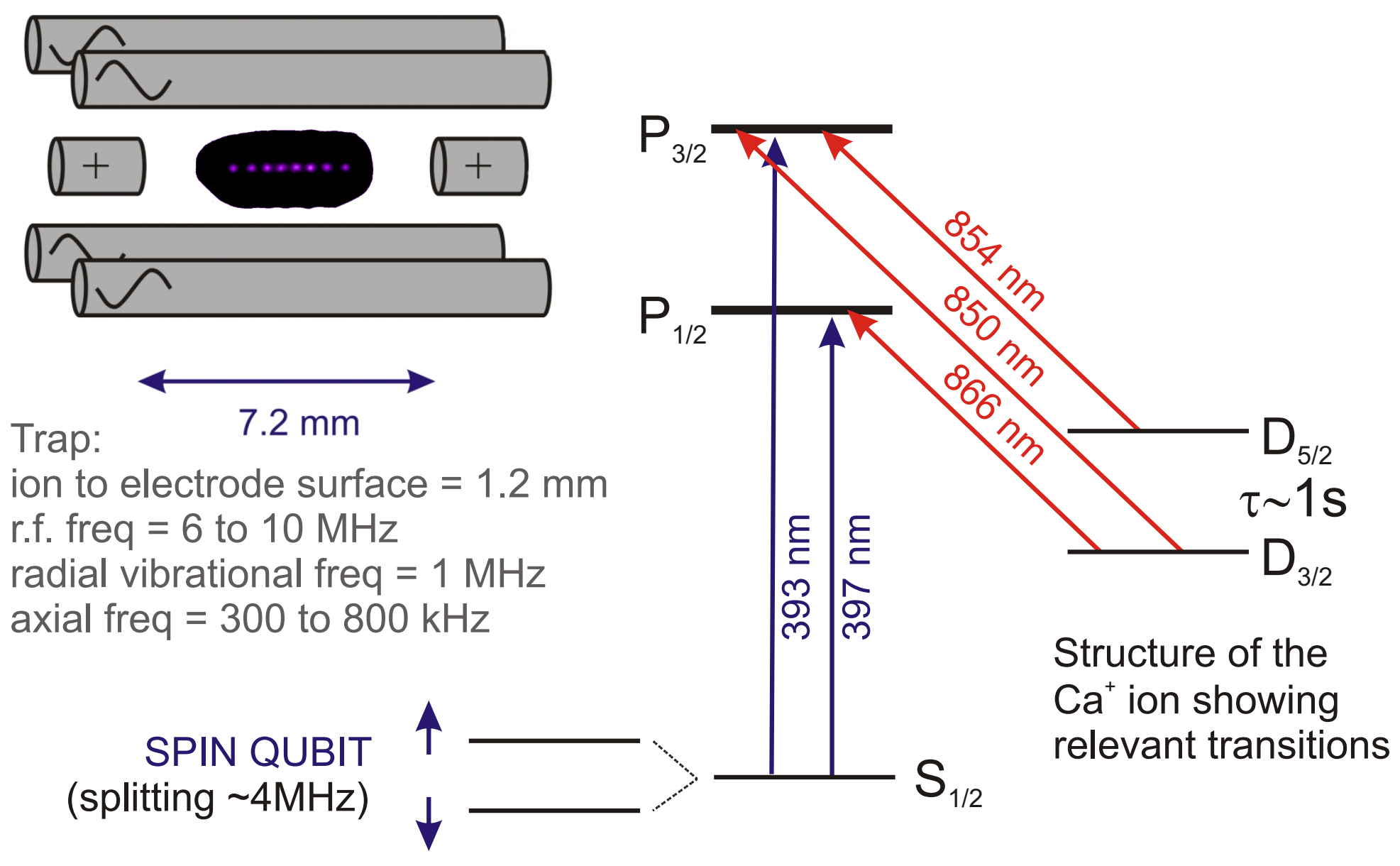
Main points

We present experiments and theory in quantum information processing using trapped ions.

The poster concentrates on entanglement and tomography experiments with calcium 40 ions: see also accompanying poster for calcium 43 work.



Trap:
ion to electrode surface = 1.2 mm
r.f. freq = 6 to 10 MHz
radial vibrational freq = 1 MHz
axial freq = 300 to 800 kHz

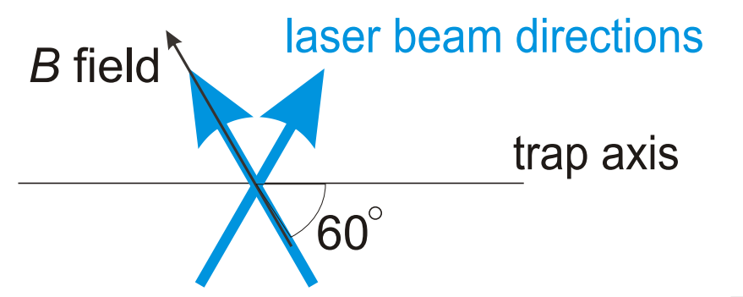
Structure of the Ca^+ ion showing relevant transitions

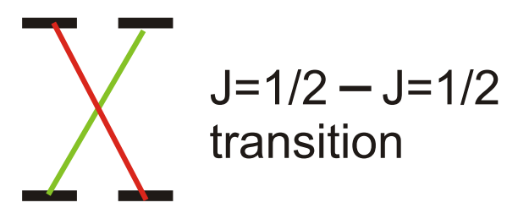
Summary of Results

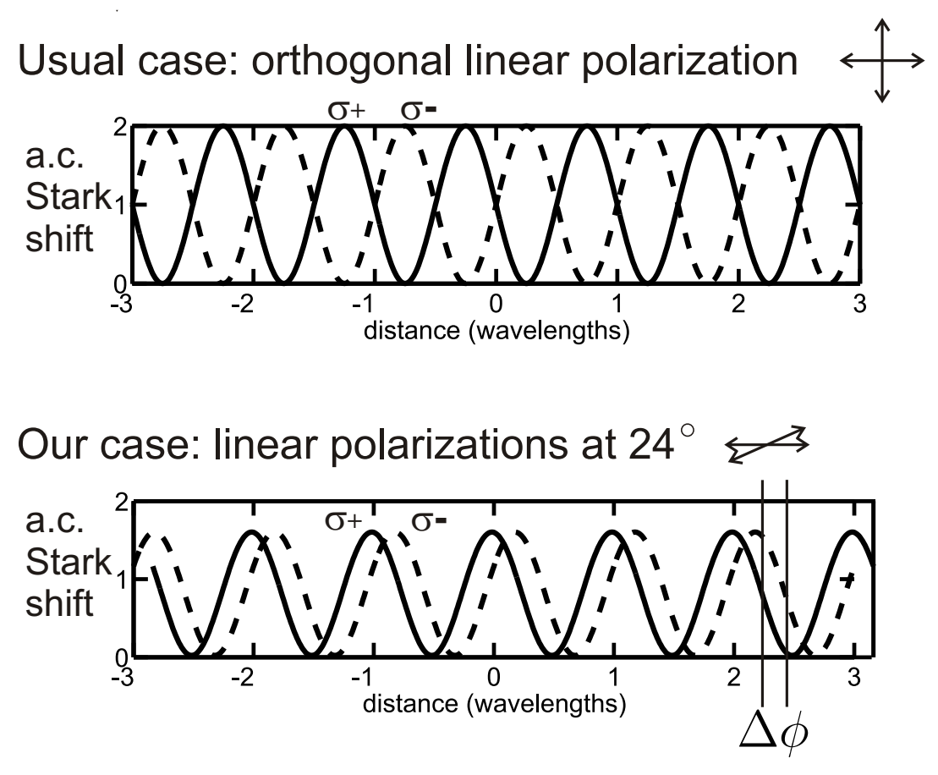
- 10 two-ion (2 qubit) Rabi flops with high visibility
- Deterministic entanglement of 2 ions (calcium 40 spin qubits) at 83(2)% fidelity
- Non Lamb-Dicke regime Schrodinger cat with 1 ion and motion:
  - α up to 4.0 ($<n>$=16)
  - $\Delta\alpha$ up to 5.4
  - well outside Lamb-Dicke regime: $\eta^2 2n = 2.1$
  - coherence time for largest cat 170 µs
  - also α = -2,0,+2 with 2 ions
- robust convenient tomography method
- 170 ms coherence time for Fock state superposition in ^{40}Ca
- 0.9 s coherence time for "clock state" qubit in ^{43}Ca.

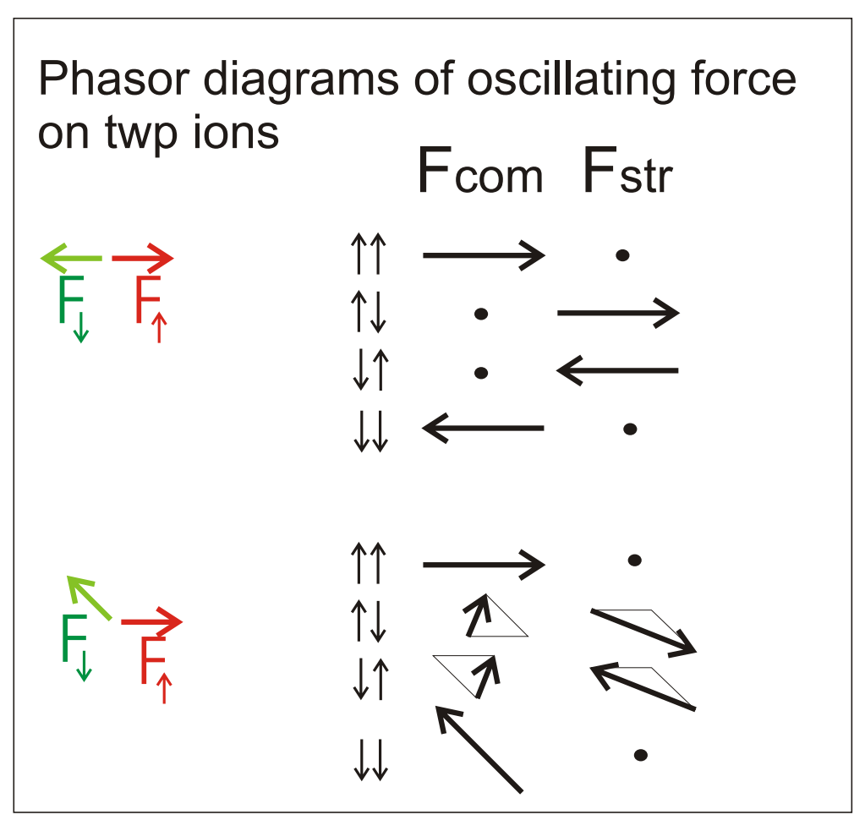
Spin-dependent force

For two-qubit gates we use spin-dependent forces: push ions depending on spin state
→ Coulomb interaction gives a two-qubit phase.
The force is an optical dipole force in a standing wave with polarization gradient.





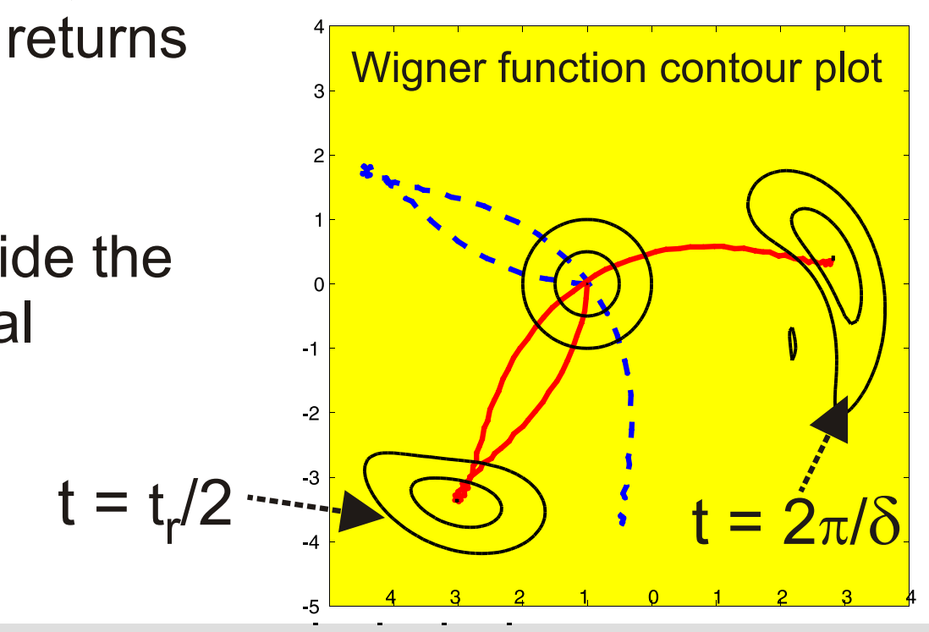




Difference frequency of laser beams ω – oscillating force on ion

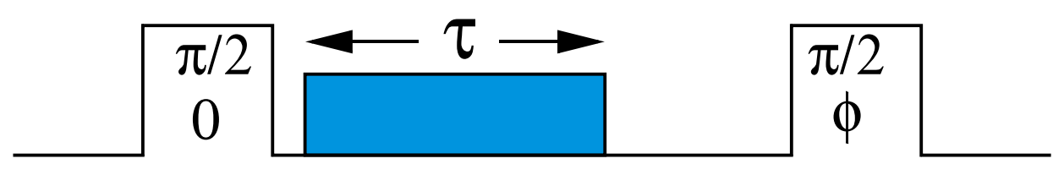
- A classical force displaces the motional state in phase space.
- In Lamb-Dicke regime, extent of motional wavepacket << λ the force appears to be spatially uniform, and an oscillating force drives the motional state around a circular loop in phase space. It returns to the origin after $t = 2\pi/\delta$, where $\delta = \omega - \omega_0$
- Outside the Lamb-Dicke regime, the trajectory of the motional state through phase space is modified, and the motional state is squeezed. The motional state returns to the origin early, ie. $t_r < 2\pi/\delta$.

We simulate the behaviour outside the Lamb-Dicke regime by numerical integration of Schrodinger's



Non Lamb-Dicke Schrödinger Cat

Coherent states of a harmonic oscillator approximate to classical motion, and a superposition of such states at mesoscopic excitation <n> is analagous to the Schrödinger's cat thought experiment.
Oscillating spin-dependent force → create such mesoscopic superpositions with single or pairs of ions. Spin state = measuring device entangled with the motion. We prove the 'cat' maintains its coherence by bringing the two parts back together and observing an interference. [As first demonstrated by Monroe *et al.* Science 272 1131

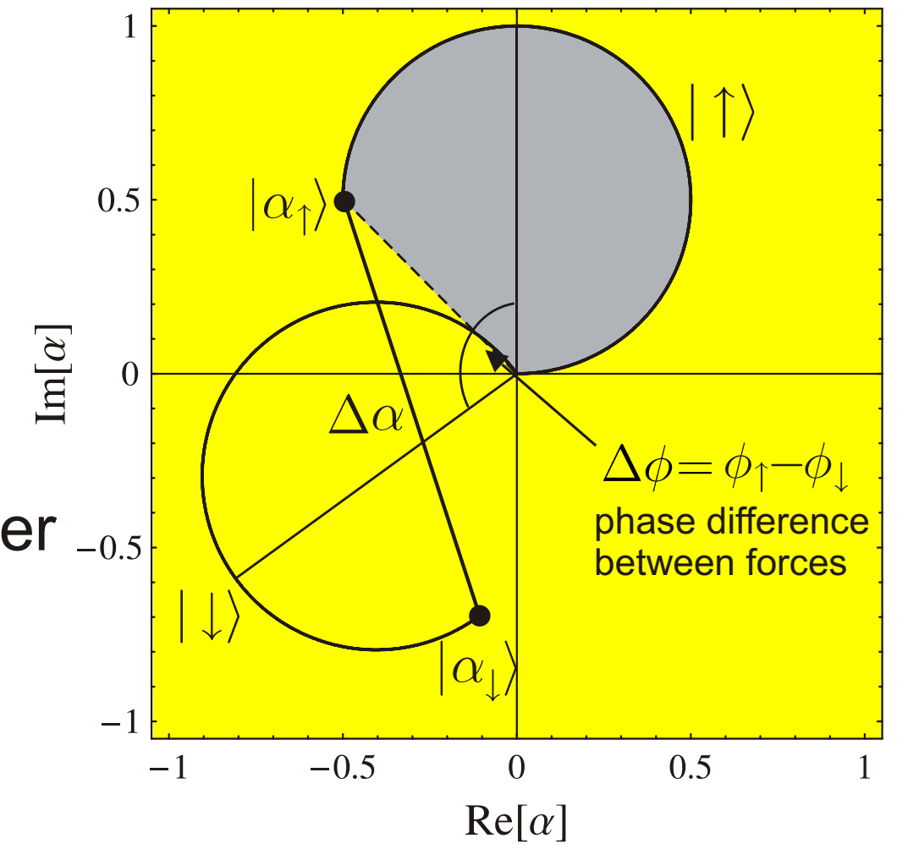


Observe P(↑) as a function of ϕ.

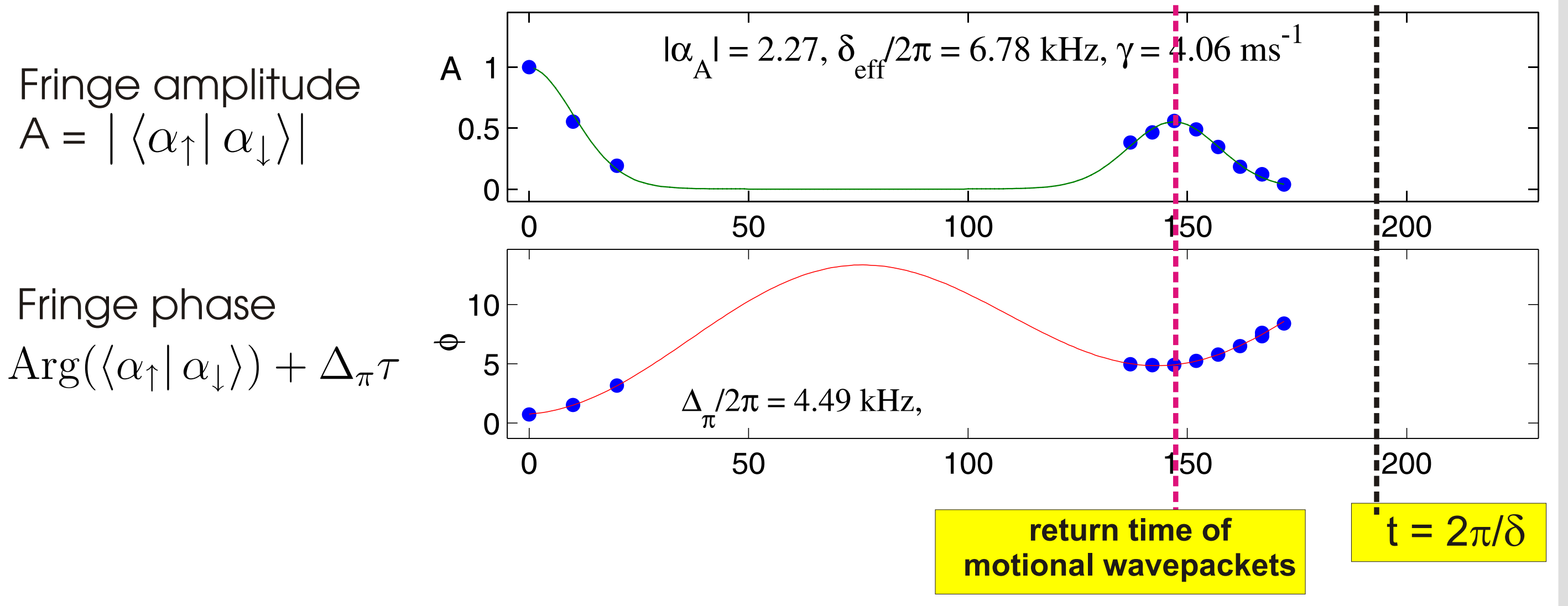
$$P(\uparrow) = \frac{1}{2}\left(1 - \mathrm{Re}\left[\langle\alpha_\uparrow|\alpha_\downarrow\rangle e^{i(\phi-\Delta_\pi\tau)}\right]\right)$$

where Δ_π is a light shift due to off-resonant carrier excitation (due to π component of the light field)

Rotating frame phase space picture (Lamb-Dicke regime)



We experimentally observe the reduction in the return time → outside L-D regime



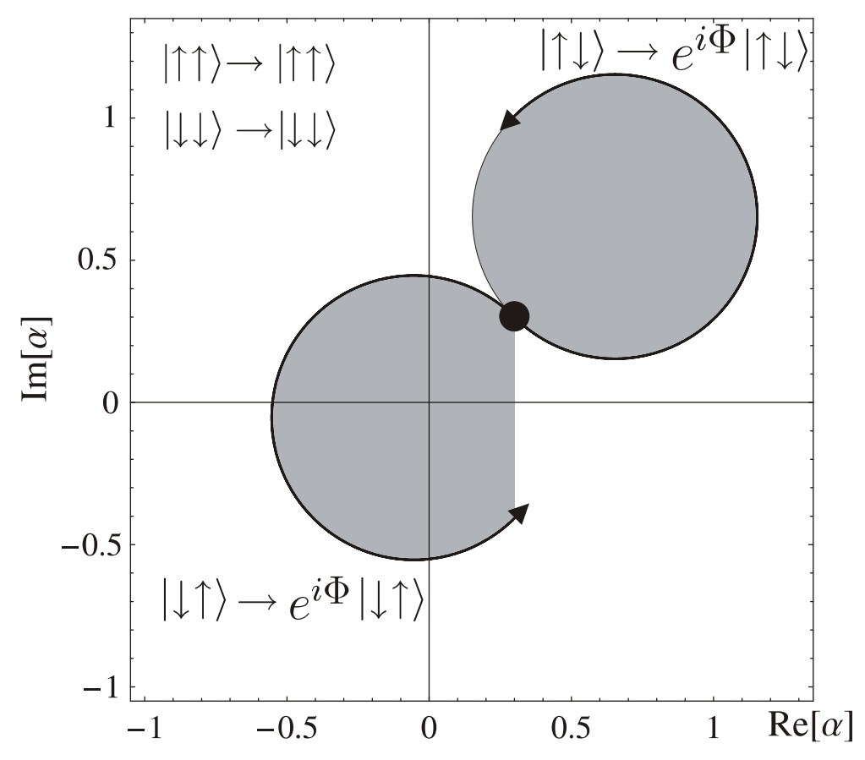
We observe cat states with $\Delta\alpha = 5.4$, $\alpha_{max} = 4$, and coherence time $T_2 = 170$ µs.

Deterministic entanglement

- Deterministic (i.e. single-shot, no post-selection) entanglement of 2 spin-qubits
- gate uses oscillating spin-dependent driving force used to create Schrödinger cats, with force frequency close to ω_{str} & ion separation = integer number of standing wave periods
  => only stretch mode excited
  => states ↑↓, ↓↑ acquire a phase; ↑↑, ↓↓ do not.
- Gate operation

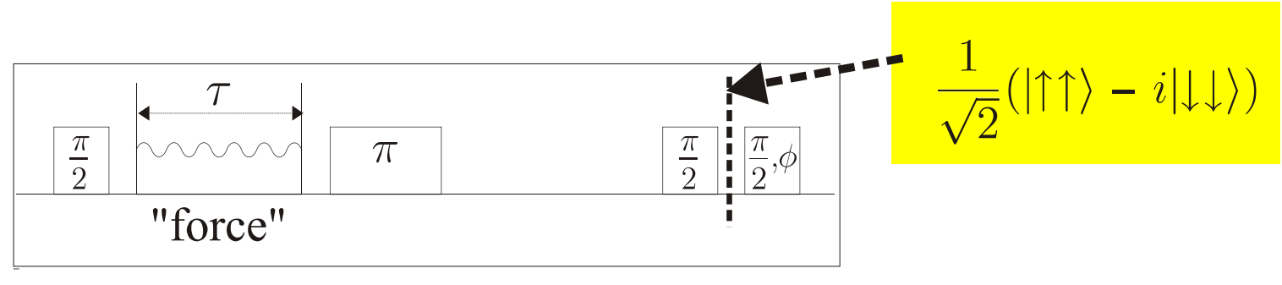
$$\hat{U} = \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & 1 \end{pmatrix}$$

(Leibfried *et al.* [Nature 422 412 (2003)].)

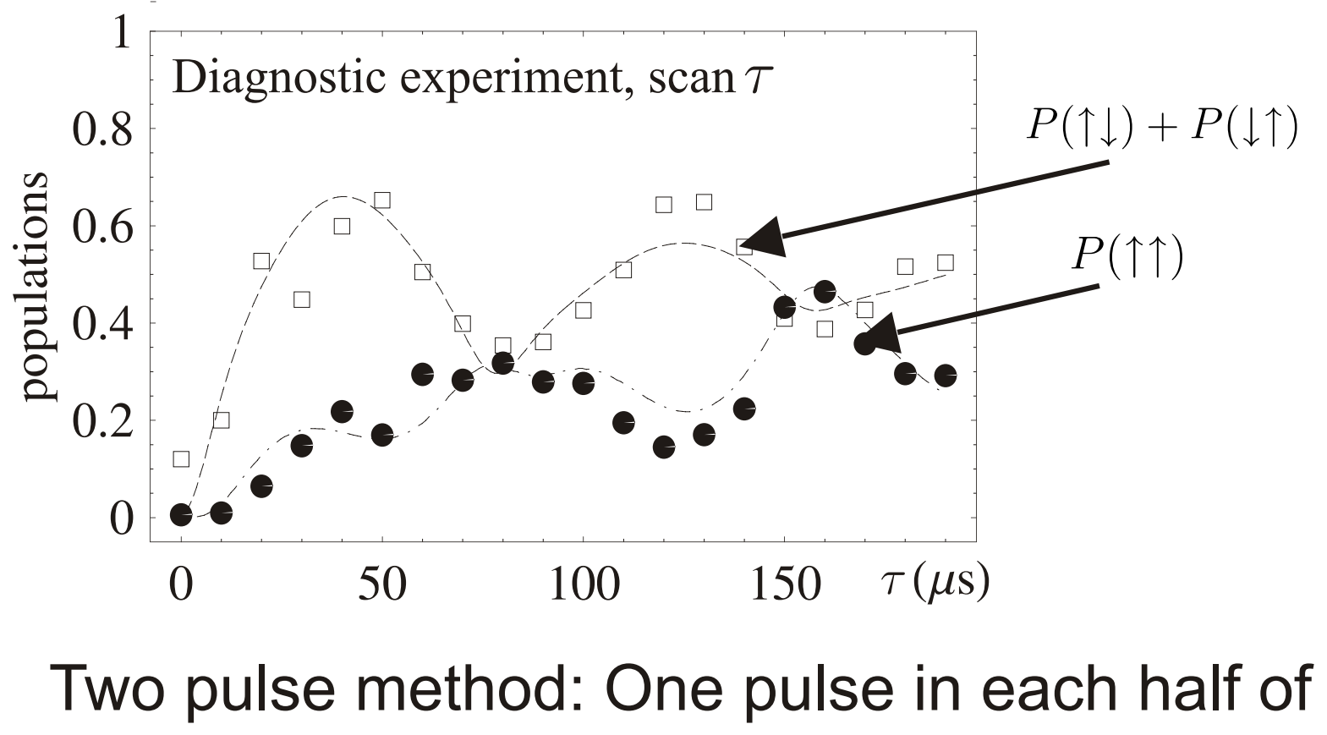


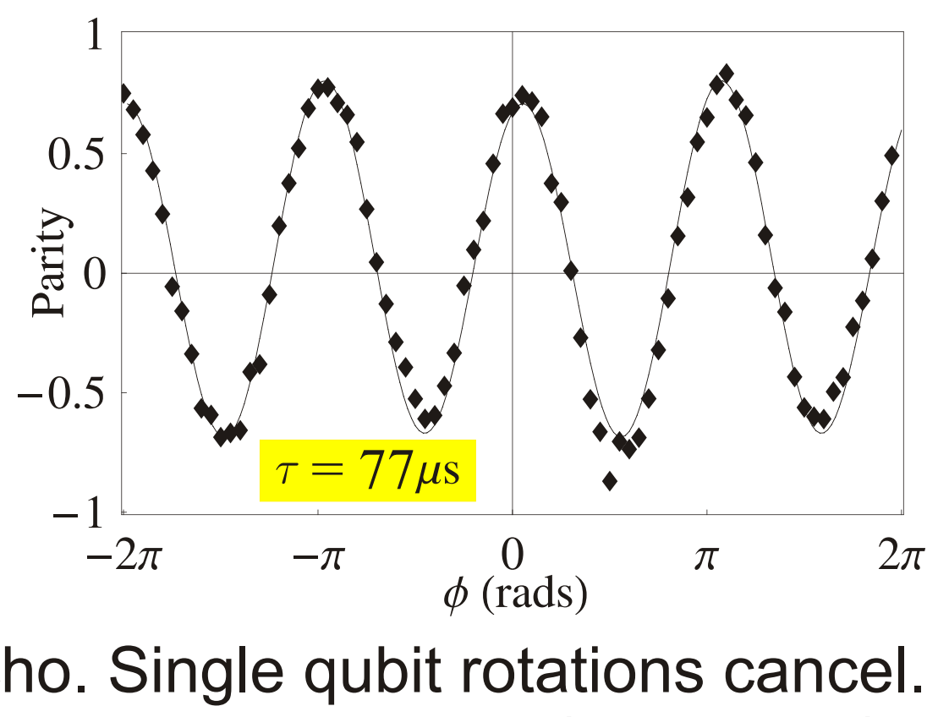
Experiments: ν_{com} = 500 kHz, ion sep = 9 µm = 22 λ

Single pulse method: Implements gate + single qubit rotations (due to light shift).
For $\Delta_\pi = 2\pi$ and $\tau = 1/\delta$, we get the entangled state.

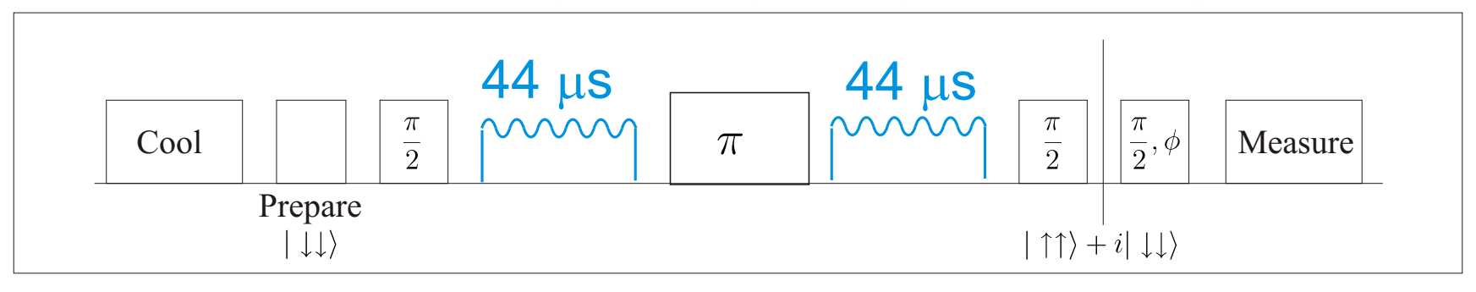


A further $\pi/2$ analysis pulse with variable phase ϕ demonstrates cos(2ϕ) oscillations in the parity signal with amplitude >0.5.
Entangled state fidelity > 74(3)%

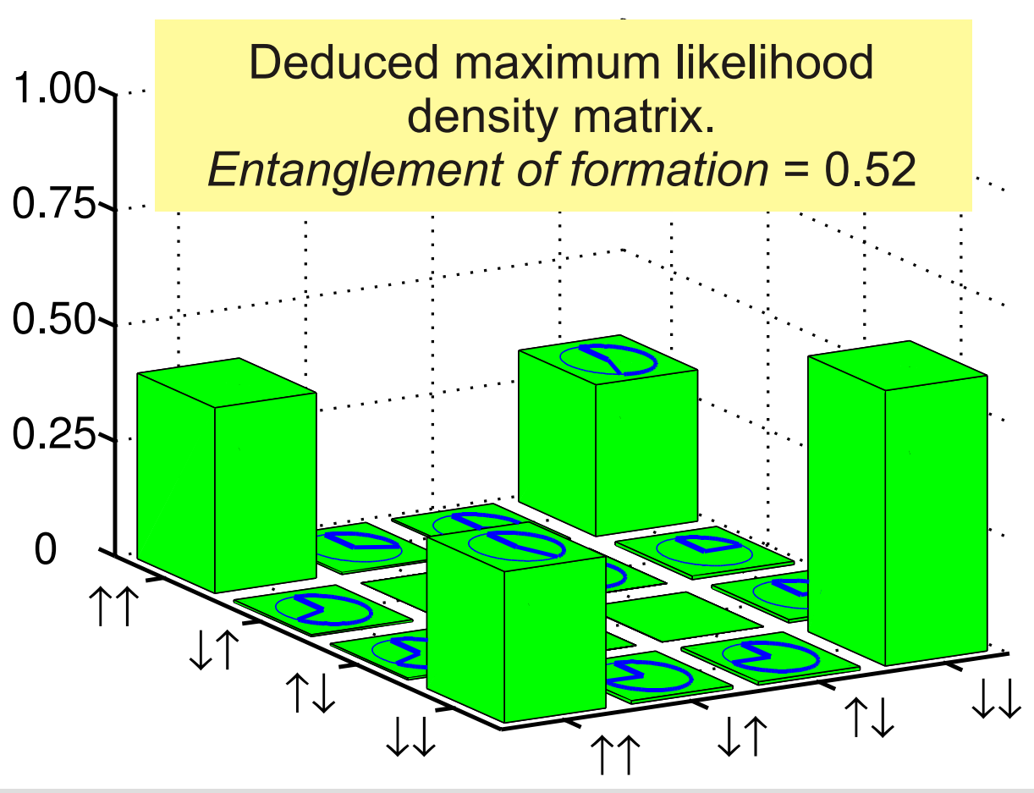




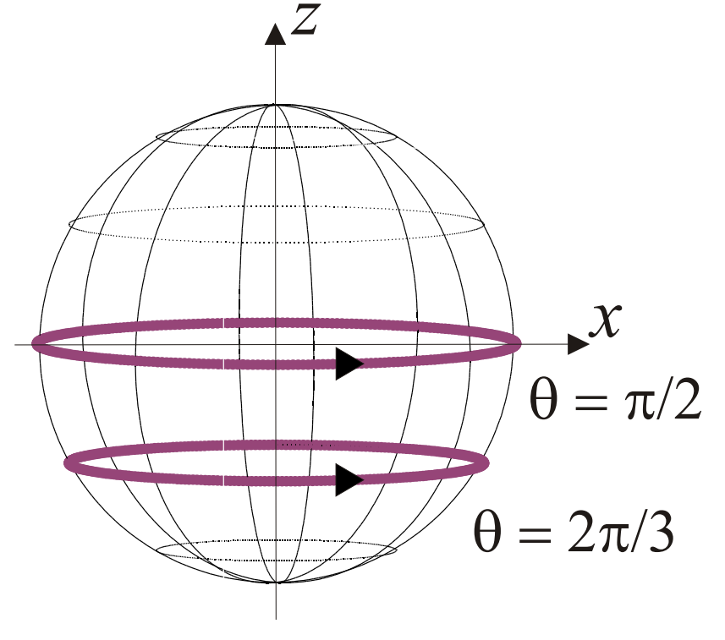
Two pulse method: One pulse in each half of spin-echo. Single qubit rotations cancel. Entangled state density matrix reconstructed using tomography method (see below). Fidelity 83(2)%



Tomography: In general, tomography involves accumulating information by applying well-chosen rotations to the qubits and measuring them in a fixed basis. We developed a convenient scheme which is robust against typical experimental issues.
The rotation is through θ about an axis ϕ in the x-y plane of the Bloch spere. ϕ is scanned from -π to π: P(spin state) =Σ(sinusoidal functions of ϕ): this allows robust curve-fitting of sin functions with period π and 2π and an offset. Each contribution to the fit yields 1 or 2 real numbers; two values of θ are needed for complete information. A maximum likelihood estimation method is then used to obtain the physical density matrix closest to that obtained from the data.


Deduced maximum likelihood density matrix.
Entanglement of formation = 0.52

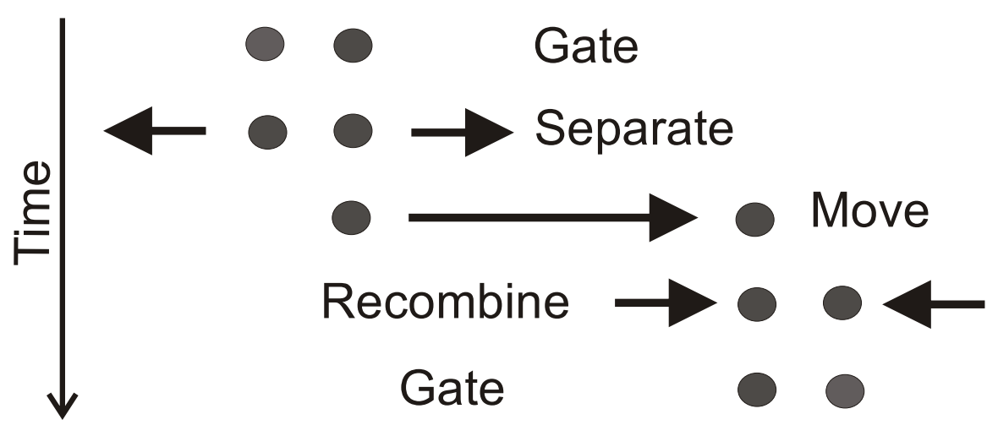
Bloch Sphere representation of rotations



(Our measurement method does not distinguish ↑↓ from ↓↑, but this has little influence for this example).

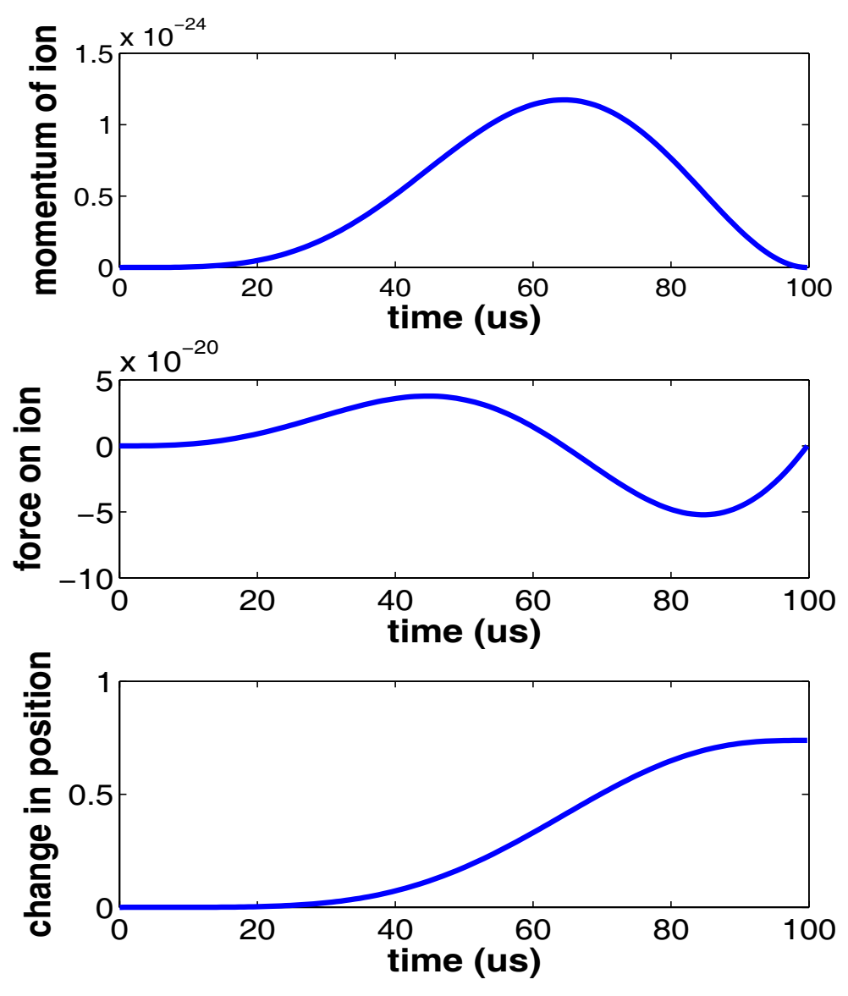
Moving ions

- Scaling up ion trap quantum computers involves using large number of ions.
- Many ions in the same trapping region → many motional modes; difficult to isolate a single mode.
- One solution: use a small number of ions in each trap and move ions around an array of traps.
  Wineland et al., J. Res. NIST 103 (1998)
  Kielpinski et al., Nature 414 (2002)



- Time scale set by trap frequency ω_z ($t >> 1/\omega_z$ for adiabatic movement)
- Move ions by changing voltages on segmented DC electrode.
- Simulate the trap potential in Charge Particle Optics (CPO) program package.

Moving an ion



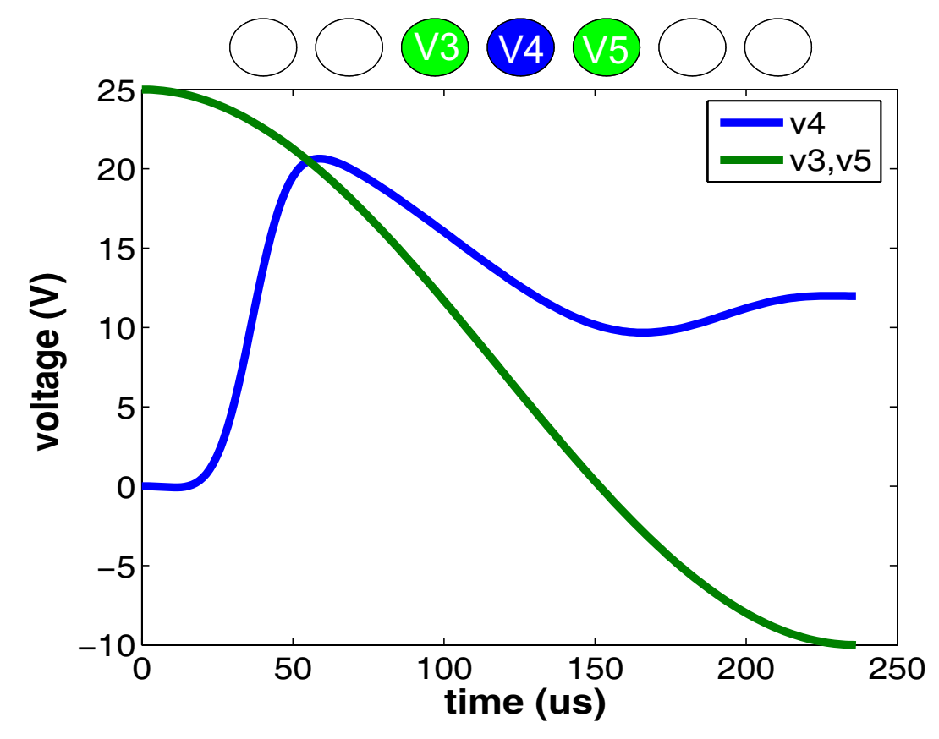
To derive the optimum electrode voltage control sequence:

- Choose a suitable function for the momentum of the ion as function of time. This defines the displacement and the force on the ion.
- Using prior simulations in CPO, deduce the voltages on the electrodes required to produce the desired force on the ion as a function of time.
- Verify the control sequence by simulating the ion's motion using Matlab.

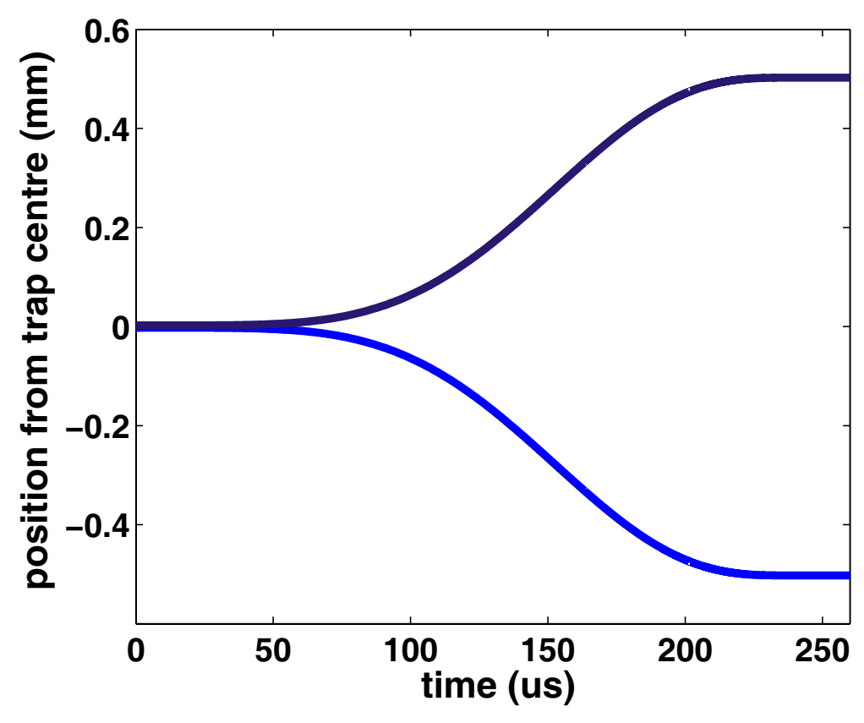
Advantages: **simple scaling** in time and final displacement. Many possible choices for momentum function.

Separating ions

Voltage control sequence



The ions' trajectory



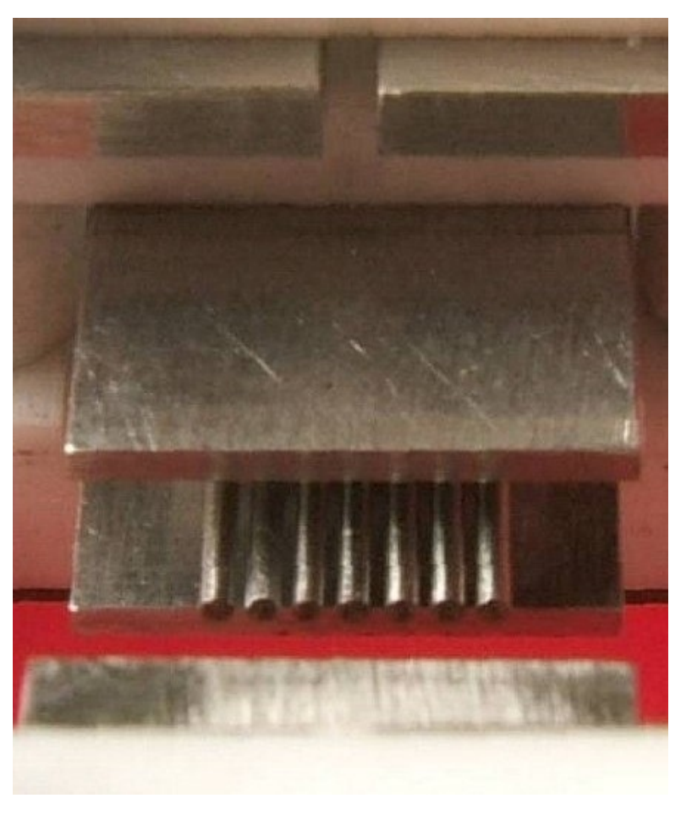
Separating ions at trap centre: Using symmetries the problem simplifies to a one ion movement process. This cannot be done when separating ions off-centre.

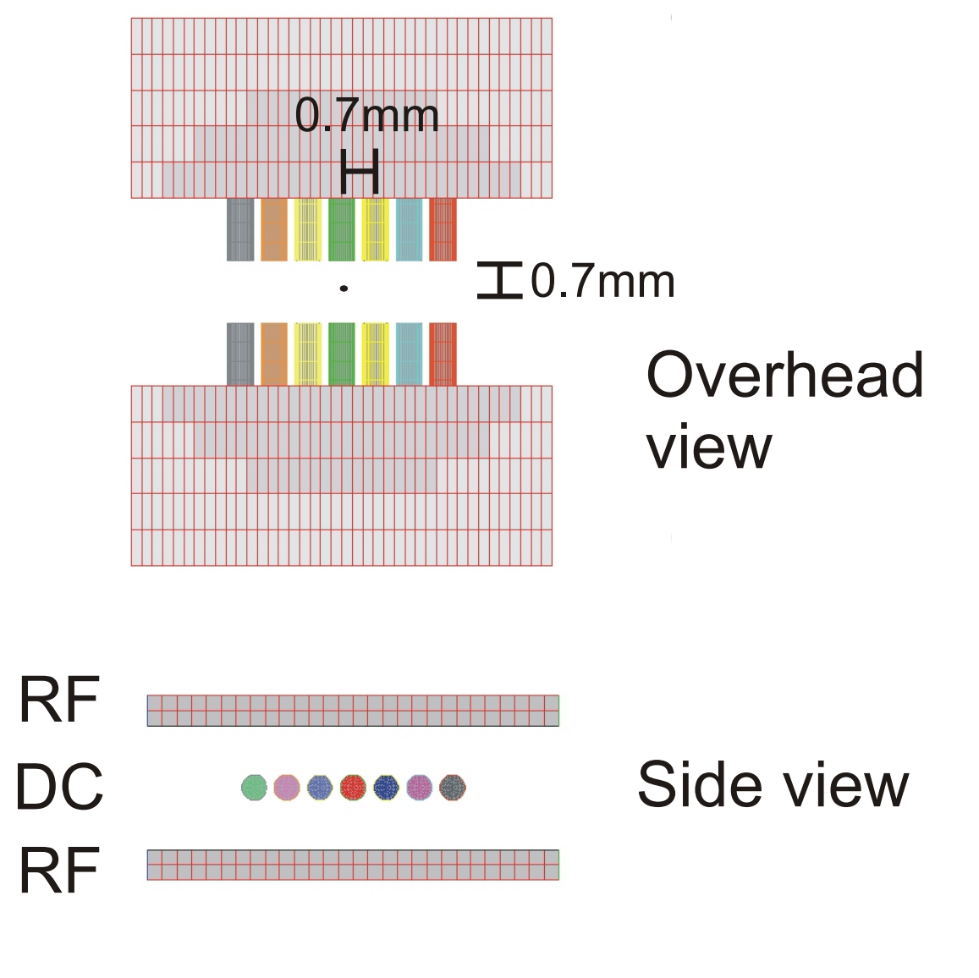
New trap designs

Macroscopic trap to study ion separation and movement:

- 7 pairs of DC electrodes, 4 RF electrodes
- Up to 3 separate trapping regions along the trap axis.
- The operating voltages were deduced by simulating the trap in CPO: RF: 1kV @ 10 MHz, DC range: (-25V,+25V)
- All RF electrodes are in phase.

Tilted view of the trap





Micron-scale trap:

- Microfabricated tungsten on silicon planar geometry.
- Scalable fabrication process.
- Mounted on chip-carrier.
- Good optical access.

Overhead view