Push and Wobble Gate; Octopole Traps

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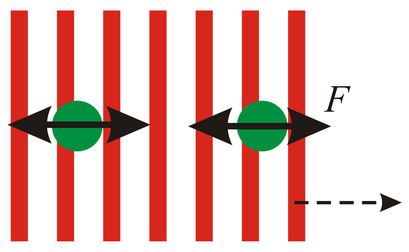
Push & Wobble Gate

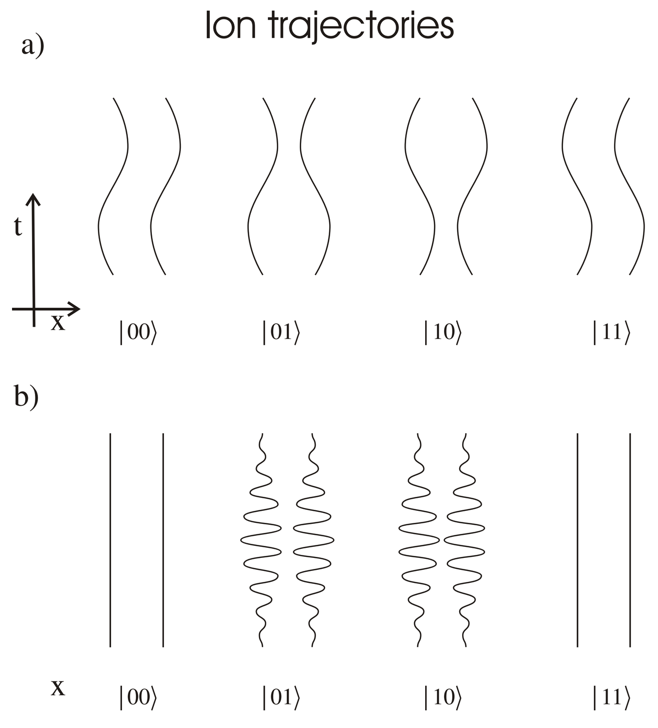
Dipole force in moving standing wave



Ion trajectories

Slow oscillation of force: "**push**"

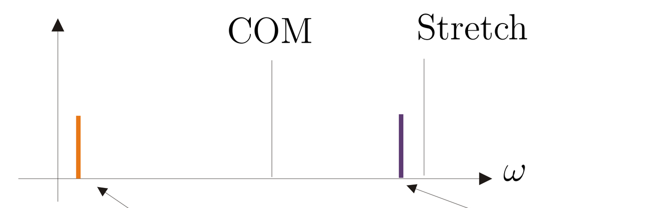
Near-resonant oscillation of force: "**wobble**"



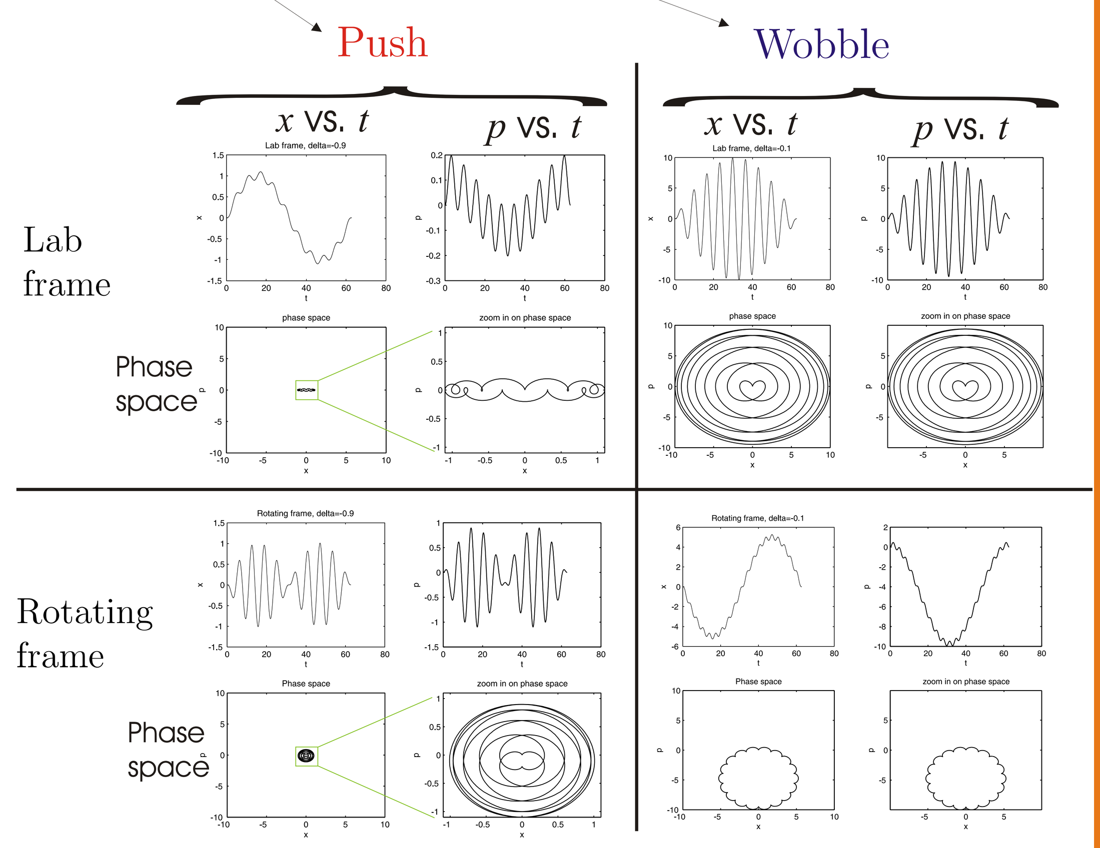
Aim: phase gate: = 0 /2 /2 0

I. Cirac and P. Zoller Nature 404 p579 (2000)
D. Liebfried et.al. Nature 422 p412 (2003)

Geometrical Argument



	Push COM	Push Str	Wobble COM	Wobble Str
00	y	-	-	-
01	-	y	-	y
10	-	y	-	y
11	y	-	-	-



Phase acquired θ = area in rotating frame

Dynamical Argument

Push: Coulomb interaction when ion pushed by distance x_F

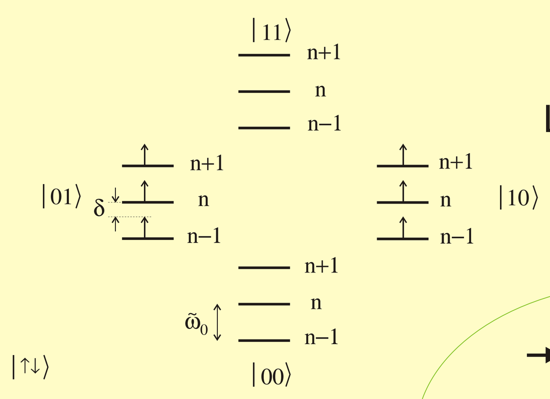
$$\begin{array}{cccc} 00 & 01 & 10 & 11 \end{array}$$

$$\frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{d}, \frac{1}{d+x_F}, \frac{1}{d-x_F}, \frac{1}{d}\right)$$

$$\simeq \frac{q^2}{4\pi\epsilon_0 d}\left(1,\ 1+\frac{x_F}{d}+\frac{x_F^2}{d^2},\ 1-\frac{x_F}{d}+\frac{x_F^2}{d^2},\ 1\right).$$

Hence $\theta_{\rm p} \simeq \frac{q^2}{4\pi\epsilon_0 d}\frac{2x_F^2}{d^2}\frac{\tau}{\hbar}.$

Wobble: Raman transition causes light shift.



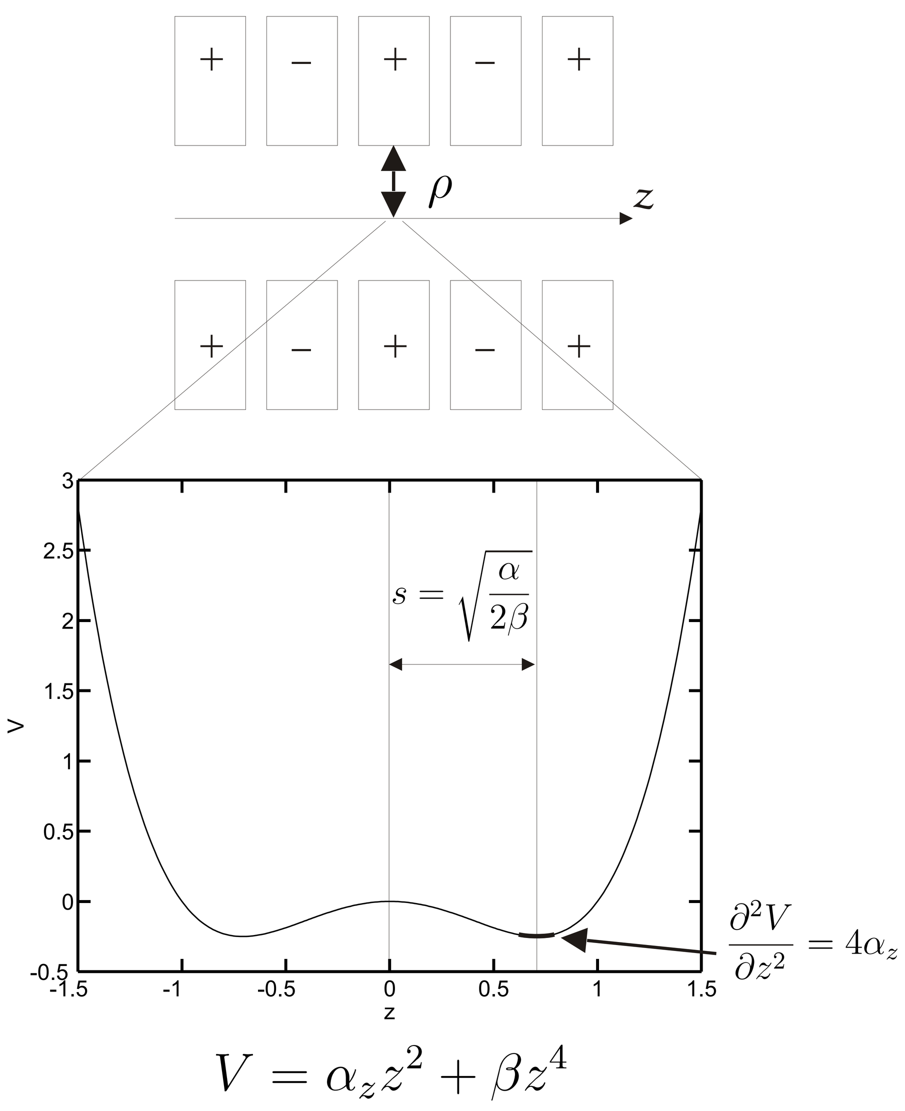
Rabi freq. $_D$ = Raman

Level n shifted up by n-1 and down by n
→ net effect ind. of n

→ $\theta_{\rm w} = 2\frac{\Omega_D^2}{\delta}\tau.$ (Choose = 2 $_D$)

c.f. $\theta_{\rm p} \simeq 16\sqrt{3}\,\frac{\Omega_D^2}{\omega_0}\tau.$

Double Well Potentials



$$V = \alpha_z z^2 + \beta z^4$$

Fidelity

Photon Scattering $N \simeq \frac{\Omega^2}{2\Delta^2}\Gamma\tau_{\rm g} = \frac{\Omega_D\Gamma\tau_{\rm g}}{\eta_L\Delta}$

Push

$$N \simeq \frac{\sqrt{3}\Gamma\omega_0}{8\eta_L\Delta\Omega_D} = \frac{\sqrt{3}\omega_0\Gamma}{4\eta_L^2\Omega_0^2}.$$

~1/intensity

Wobble

$$N \simeq \frac{\pi\Gamma}{\eta_L\Delta}$$

~1/detuning

Thermal

	Push	Wobble
Non-uniform force = deviation from Lamb-Dicke approx	$P \simeq 2\pi^2\eta_L^4\left(\frac{k_BT}{\hbar\omega_0}\right)^2$	$P = \eta_L^4(\pi^2/4)\bar{n}(\bar{n}+1)$
Coupling to other mode	—	$P \simeq 1.6\frac{\Omega_D^2}{\tilde{\omega}_0^2}(\bar{n}+1).$
Debye-Waller factor		$P = \eta_L^4(\pi^2/4)\,0.4\,\bar{n}(\bar{n}/2+1)$

The total thermal effect is similar for the two cases: $P \simeq (\bar{n}+1)\left(0.3\pi^2\eta_L^4\bar{n} + 1.6\frac{\Omega_D^2}{\tilde{\omega}_0^2}\right)$

Laser intensity noise: $\sim I$ hence $P \sim {}^2 \sim I^2$

M.Sasura and A.M.Steane, PRA 62 062318 (2003)
A.Sorenson and K.Molmer PRA 62 022311 (2000)

Discussion

Gate time = 2 (Push) or 2 / (Wobble)

Wobble uses resonance to displace ions further for a given force --> requires less laser intensity (for given scattering N).

Laser intensity noise typically falls vs. frequency --> better to oscillate the force at high freq. to cancel single-bit phases.

However, for a faster gate require not too small
--> intermediate regime where both modes are excited.

Fine structure --> limit on
-->

$$N = \frac{2\sqrt{2}\,\pi\Gamma}{\eta\omega_{\rm F}} = \frac{2(\lambda\Gamma)\sqrt{m\omega_z/\hbar}}{\omega_{\rm F}}$$

but $_{\rm F} \sim m^2$ so $_z \sim m^3 N^2$

e.g. Ca 43 vs. Ba 137 : factor 30 in speed or 6 in N

D.C. Octopole

Close wells ⇒ large β
This tends to imply large $|\alpha_x|$, $|\alpha_y|$, $|\alpha_z|$
But to confine we require small $|\alpha_x|$, $|\alpha_y|$
⇒ Octopole + R.F. Quadrupole + small D.C. Quadrupole

$$V(x,y,z,t) \simeq \alpha\left(z^2 - \frac{1}{2}(x^2+y^2)\right) + \beta V_4(x,y,z) + Q_{\rm ac}\cos(\Omega t)(x^2-y^2)$$

Geometric factors γ, μ defined by

$$\text{for } Q_{\rm ac}=0,\quad \beta = \frac{\gamma E_{\rm max}}{\rho^3},$$

$$\text{for } \beta=0,\quad Q_{\rm ac} = \frac{\mu E_{\rm max}}{\rho}.$$

($E_{\rm max}$: Maximum electric field; ρ: Distance to nearest surface)

For two ions;

$$\omega_{\rm COM}^2 = (2\alpha_z + 3\beta d^2)q/m$$
$$\omega_{\rm stretch}^2 = \omega_{\rm COM}^2(1+\tilde{\epsilon})$$

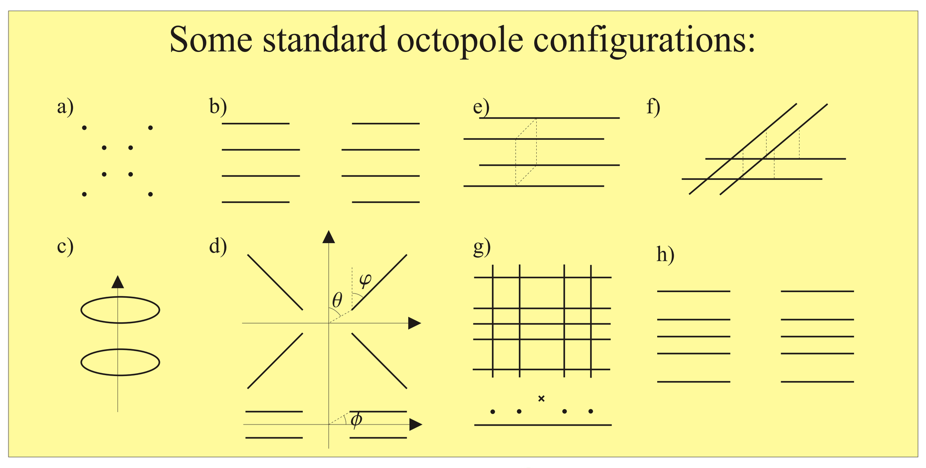
where

$$\tilde{\epsilon} = \frac{q^2}{\pi\epsilon_0 m\omega_{\rm COM}d^3}$$

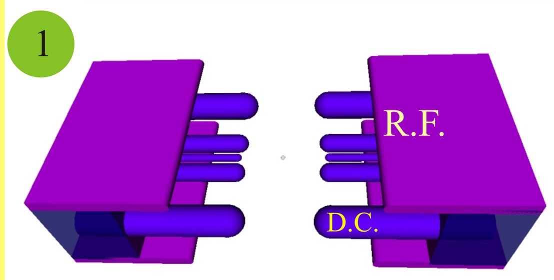
d = separation of the ions.

Electrode Configurations

Some standard octopole configurations:

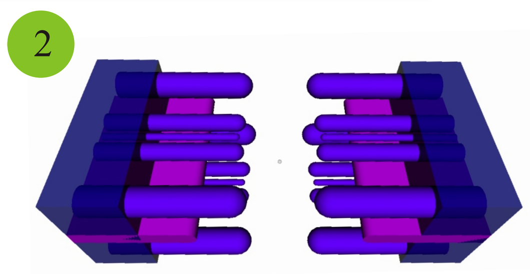


1



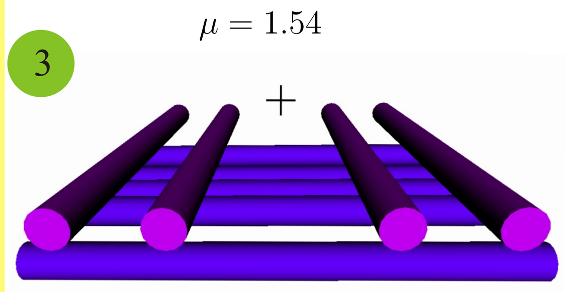
$\gamma = 0.027$
$\mu = 1.54$

2



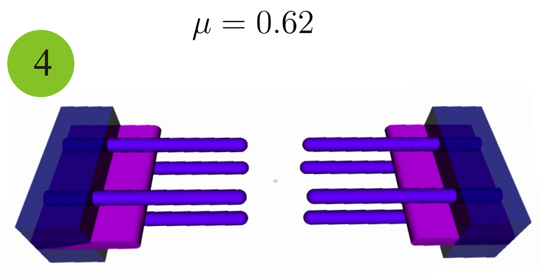
$\gamma = 0.0065$
$\mu = 0.62$

3



$\gamma = 0.0064$
$\mu = 0.48$

4



$\gamma = 0.0033$
$\mu = 0.072$

Trap frequencies vs ρ

$$\frac{\omega_{\rm COM}(\text{octopole})}{2\pi}[\text{MHz}] \simeq \frac{840}{\sqrt{A}}\frac{(\gamma E_{\rm max})^{3/10}}{\rho^{9/10}}.$$

$$\frac{\omega_{\rm radial}}{2\pi}[\text{MHz}] \simeq \frac{1105}{\sqrt{A}}\left(\frac{q_r\mu E_{\rm max}}{\rho}\right)^{1/2}$$

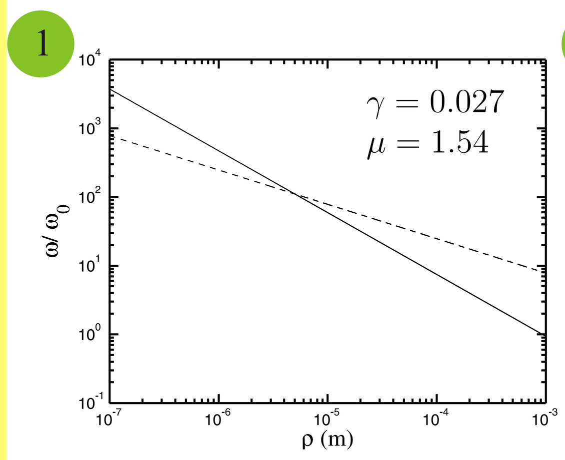
A = mass no. of ion, q_r = Mathieu q parameter
$E_{\rm max}$ in Vμm^{-1}, ρ in μm.

ρ small; $\omega_{\rm radial} < \omega_{\rm COM}$(octopole)
ρ large; $\omega_{\rm radial} > \omega_{\rm COM}$(octopole)

Define $\omega_0 = (\omega_{\rm radial}$ for $Q_{\rm ac} = 10^6$V/m$^2)$.
At $E_{\rm max} = 10^6$V/m, $q_r = 0.3$.

Ion	$\omega_0/2\pi$ (kHz)
^{9}Be	201.8
^{25}Mg	121.1
^{43}Ca	92.3
^{67}Zn	73.9
^{87}Sr	64.9
^{111}Cd	57.5
^{135}Ba	52.1
^{171}Yb	46.3
^{199}Hg	42.9

1



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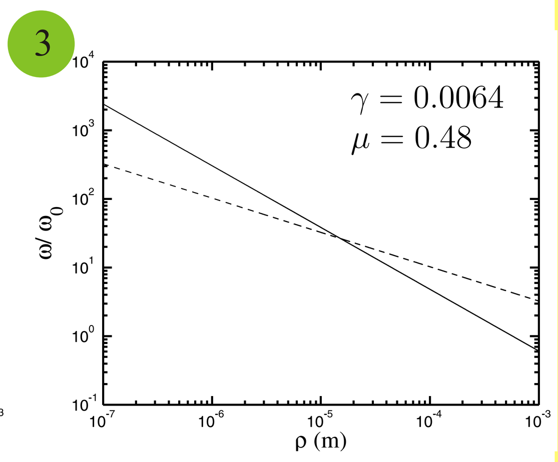

$\omega_{\rm COM}$(octopole) ———— $\omega_{\rm radial}$ - - - - - - -